MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor A. S. EDDINGTON, F.R.S., PRESIDENT, in the Chair.

Miss Frances Evelyn Cave-Browne-Cave, M.A., Director of Mathematical Studies, Girton College, Cambridge;

William Michael Herbert Greaves, B.A., Smith's Prizeman, St. John's College, Cambridge;

Richard Griffith Quirk, Principal of the Dublin School of Navigation and Seamanship, 4 Eglinton Park and 23 Eden Quay, Dublin; and

John Whitridge Wilson, 87 Woodvale, Forest Hill, London, S.E. 23,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Rev. Tegfan Davies, D.D., The Manse, New Road, Ammonford, Carmarthenshire (proposed by John Rees);

Thomas George Hirst, High Barn, Birstal, near Leicester (proposed by J. W. Durrad);

Rev. Urban Lindsay Johnstone, M.A., Aston Vicarage, Sutton Weaver, near Warrington (proposed by Walter Bidlake);

Gerald Harper Lepper, 40 Radnor Road, Harrow, Middlesex (proposed by P. J. Melotte);

Frederick William Lutjens, Chief Officer, s.s. *Cranmore*, 80 Lansdowne Road, Tottenham, London, N. 17 (proposed by W. R. Cullington);

William Duncan MacMillan, Professor of Astronomy in the University of Chicago (proposed by Ludwik Silberstein);

Walter Allen Moss, 11 Tressillian Crescent, Brockley, London, S.E. 4 (proposed by Percy H. Davis);

Professor Abel Pérez Sánchez, Colonia 1139, 3$^{er\ piso}$, Montevideo, Uruguay (proposed by Henry Ellis);

William Smalley, Springfield, Castleton, near Manchester (proposed by A. L. Cortie);
William John Thorrowgood, M.I.E.E., M.I.R.S.E., 2 Waldemar Road, Wimbledon Park, London, S.W. 19 (proposed by E. W. Maunder);
Harihar Parames Waran, M.A., Christ's College, Cambridge (proposed by A. S. Eddington); and
Dr. Jan Woltjer, Conservator at the Observatory of Leiden, Holland (proposed by W. de Sitter).

One hundred and sixty-five presents were announced as having been received since the last meeting, including, amongst others:

Drawing of Comet of 1843, presented by Miss A. Breton.
Cape of Good Hope: 2nd Fundamental Catalogue of Stars for the equinox 1900, 1912–16.
M. Dumersan: Le Zodiaque de Dendéra, presented by Mr. Inwards.
Royal Observatory, Greenwich: Catalogue of Double Stars and other publications.
San Fernando: Astrographic Chart, 42 maps.
Royal Observatory, Belgium: Astrographic Chart, 40 maps.
Hyderabad, Nizamiah Observatory: Astrographic Catalogue, vol. iv.
Helwân Observatory: Eleven slides of Nebulæ.

Note on the Path of a Ray of Light in the Einstein Relativity Theory of Gravitational Effect. By Professor A. R. Forsyth, F.R.S., Imperial College of Science and Technology, London, S.W.

The Accurate Equation of the Path.

1. The critical equation in the Einstein theory for the gravitational deflection of a ray of light is

$$\frac{1}{\gamma}\left(\frac{dr}{dt}\right)^2+\left(r\frac{d\phi}{dt}\right)^2=\gamma,$$

where

$$\gamma=1-\frac{2m}{r}.$$

To obtain the path itself, Huygens' principle—that the time between two points is stationary for small variations of the path—is of immediate service. We have

$$\left(\frac{dt}{dr}\right)^2=\frac{r^2}{\gamma}\left(\frac{d\phi}{dr}\right)^2+\frac{1}{\gamma^2},$$

so that

$$t=\int\left\{\frac{r^2}{\gamma}\left(\frac{d\phi}{dr}\right)^2+\frac{1}{\gamma^2}\right\}^{\frac{1}{2}}dr$$

$$=\int \mathrm{U}dr.$$

The mathematical expression of the principle is

$$\delta t = 0 ;$$

the customary process leads to the equation

$$\frac{d}{dr}\left(\frac{\partial U}{\partial \phi'}\right) - \frac{\partial U}{\partial \phi} = 0,$$

where ϕ' denotes $d\phi/dr$. As U does not explicitly involve ϕ, we have

$$\frac{\partial U}{\partial \phi'} = \text{constant} = E,$$

so that

$$\frac{1}{U}\frac{r^2}{\gamma}\frac{d\phi}{dr} = E ;$$

and consequently

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4}{E^2} - r^2 + 2mr.$$

Let D denote the apsidal distance, so that

$$\frac{D^3}{E^2} - D + 2m = 0 ;$$

and let the angle ϕ be measured positively from the apsidal direction, so that r and ϕ increase together. Now substitute

$$r = \frac{1}{u} ;$$

then

$$\begin{aligned} d\phi &= -\frac{1}{(2m)^{\frac{1}{2}}}\left(u^3 - \frac{u^2}{2m} + \frac{1}{2mE^2}\right)^{-\frac{1}{2}} du \\ &= -\frac{1}{(2m)^{\frac{1}{2}}}\left(u^3 - \frac{u^2}{2m} + \frac{D - 2m}{2mD^3}\right)^{-\frac{1}{2}} du \\ &= -\frac{1}{(2m)^{\frac{1}{2}}}\{(u-\alpha)(u-\beta)(u-\gamma)\}^{-\frac{1}{2}} du, \end{aligned}$$

where

$$\alpha = \frac{1}{4mD}\{D - 2m + (D^2 + 4Dm - 12m^2)^{\frac{1}{2}}\},$$

$$\beta = \frac{1}{D},$$

$$\gamma = \frac{1}{4mD}\{D - 2m - (D^2 + 4Dm - 12m^2)^{\frac{1}{2}}\}.$$

The quantities $1/\alpha$, $1/\beta$, $1/\gamma$ are possible apsidal distances. But $1/\alpha$ is approximately $2m$; the path would pass nearly through the centre of the sun, so that $1/\alpha$ is not an effective apsidal distance. Also, as γ is negative, $1/\gamma$ must be rejected. The only effective apsidal distance is D.

To find the value of ϕ, let

$$u = \gamma + (\beta - \gamma)\sin^2\theta,$$

$$k^2 = \frac{\beta - \gamma}{\alpha - \gamma};$$

then

$$d\phi = -\frac{1}{(2m)^{\frac{1}{2}}}\frac{2}{(\alpha-\gamma)^{\frac{1}{2}}}\frac{d\theta}{(1-k^2\sin^2\theta)^{\frac{1}{2}}}.$$

In the path between the star and perihelion, u ranges from 0 to $\frac{1}{D}$, that is, from 0 to β; for the latter value, $\theta = \frac{1}{2}\pi$. If $\phi = \varpi$ at perihelion, we have

$$\phi - \varpi = -2\rho\int_{\frac{1}{2}\pi}^{\theta}(1-k^2\sin^2\theta)^{-\frac{1}{2}}d\theta$$

$$= 2\rho(\mathrm{K} - \mu),$$

where

$$\rho = \{2m(\alpha-\gamma)\}^{-\frac{1}{2}},$$

$$\theta = \mathrm{am}\,\mu,$$

and K is the complete elliptic integral of the first kind, that is,

$$\theta = \mathrm{am}\left(\mathrm{K} - \frac{\phi - \varpi}{2\rho}\right).$$

Thus

$$u = \gamma + (\beta - \gamma)\sin^2\theta$$

$$= \gamma + (\beta - \gamma)\,\mathrm{sn}^2\left(\mathrm{K} - \frac{\phi - \varpi}{2\rho}\right),$$

or

$$\frac{1}{r} = \gamma + (\beta - \gamma)\frac{1 + \mathrm{cn}\dfrac{\phi - \varpi}{\rho}}{1 + \mathrm{dn}\dfrac{\phi - \varpi}{\rho}},$$

which is the accurate (not approximate) equation of the path, on the assumption that the original critical equation is accurate.

The Path is everywhere Concave to the Sun.

2. The equation of the path is

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4}{D^3}(D - 2m) - r^2 + 2mr,$$

so that

$$\frac{d^2r}{d\phi^2} = 2\frac{r^3}{D^3}(D - 2m) - r + m.$$

Hence the radius of curvature of the path, being

$$\frac{\left\{r^2+\left(\frac{dr}{d\phi}\right)^2\right\}^{\frac{3}{2}}}{r^2+2\left(\frac{dr}{d\phi}\right)^2-r\frac{d^2r}{d\phi^2}}$$

for any curve, becomes

$$\frac{r^{\frac{1}{2}}}{3m}\left\{\frac{r^4}{\mathrm{D}^3}(\mathrm{D}-2m)+2mr\right\}^{\frac{3}{2}}.$$

Because this is positive, the path is everywhere concave to the sun.

The smallest value of r is D. Having regard to the actual numbers, we see that the path is very slightly curved inwards to the sun at perihelion, and that, away from perihelion, the curvature is exceedingly slight.

The Co-ordinate Velocity in the Path.

3. As regards the co-ordinate velocity v, we have

$$v^2=\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\phi}{dt}\right)^2,$$

while

$$\gamma^{-1}\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\phi}{dt}\right)^2=\gamma.$$

Hence

$$v^2=\gamma^2\frac{\left(\frac{dr}{d\phi}\right)^2+r^2}{\left(\frac{dr}{d\phi}\right)^2+\gamma r^2}$$

$$=\gamma^2\left(1+\frac{2m}{\mathrm{D}-2m}\frac{\mathrm{D}^3}{r^3}\right),$$

and therefore

$$v=\gamma\left(1+\frac{2m}{\mathrm{D}-2m}\frac{\mathrm{D}^3}{r^3}\right)^{\frac{1}{2}}.$$

It is to be noted that, in obtaining this result, no approximation has been used.

At perihelion, the magnitude of the co-ordinate velocity is

$$\left(1-\frac{2m}{\mathrm{D}}\right)^{\frac{1}{2}}.$$

The Deflection of a Ray passing from a Star to the Earth: its General Value.

4. To estimate the total deflection of a ray in passing from a star to the earth as it goes near the sun, it is convenient to divide the path into two portions, one on each side of the perihelion of the path. And, even before approximations are made, it is desirable to have a notion of the

actual numbers, though they are not introduced at this stage of the calculations.

Let R denote the radius of the sun; R_2 the distance of the earth from the sun; R_1 the distance of the star Piazzi IV, 82, from the sun.* Then we have, as working measures,

$$\frac{R_2}{R} = 2{\cdot}15 \times 10^2, \qquad \frac{R_1}{R} = 1{\cdot}63 \times 10^9.$$

Thus, for the practicably accurate range of observation, R/R_2 is a finite number though it is small, while R/R_1 is a very small number.

Consider the first portion of the path between the star and perihelion. At the star itself, let χ_1 be the angle between the solar radius vector and the tangent to the path; thus

$$\tan \chi_1 = \text{value of } r\frac{d\phi}{dr} \text{ when } r = R_1$$

$$= \frac{R_1}{\left\{\frac{R_1^4}{D^3}(D - 2m) - R_1^2 + 2mR_1\right\}^{\frac{1}{2}}}.$$

(Owing to the magnitude of R_1, the magnitude of χ_1 is negligible in comparison with practical measurements; so that the solar radius vector to the star effectively gives the direction of the emanating light. But this consideration can be deferred for the moment.)

At the star, the value of θ is given by

$$\frac{1}{R_1} = \gamma + (\beta - \gamma)\sin^2\theta_1;$$

and (again deferring the consideration of the very large value of R_1) we have

$$\phi_1 - \varpi = 2\rho(K - \mu_1),$$

where μ_1 is given by the relation

$$\theta_1 = \text{am}\,\mu_1.$$

At perihelion, the direction of the ray is perpendicular to the solar radius vector; at the star, the inclination of the ray to the solar radius vector is

$$(\phi_1 - \varpi) + \chi_1$$
$$= 2\rho(K - \mu_1) + \chi_1.$$

Hence the deflection of the ray, in passing from the star to perihelion, is

$$2\rho(K - \mu_1) + \chi_1 - \tfrac{1}{2}\pi.$$

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* *Phil. Trans.*, Ser. A, 220 (1920), p. 293. This star is selected as giving the smallest value for R_1.

I wish to take this opportunity of thanking the Astronomer Royal, Sir Frank Dyson, for his kindness in providing me with numerical data.

Similarly, on the other side of the perihelion and in the same sense, the deflection of the ray in passing from perihelion to the earth is

$$2\rho(\mathrm{K}-\mu_2)+\chi_2-\tfrac{1}{2}\pi,$$

where

$$\tan\chi_2 = \text{value of } r\frac{d\phi}{dr} \text{ when } r=\mathrm{R}_2$$

$$=\frac{\mathrm{R}_2}{\left\{\frac{\mathrm{R}_2^4}{\mathrm{D}^3}(\mathrm{D}-2m)-\mathrm{R}_2^2+2m\mathrm{R}_2\right\}^{\frac{1}{2}}},$$

where μ_2 is given by the relation

$$\theta_2 = \mathrm{am}\mu_2,$$

and the value of θ_2 is given by

$$\frac{1}{\mathrm{R}_2}=\gamma+(\beta-\gamma)\sin^2\theta_2,$$

R_2 being the radial distance of the earth from the sun.

We take ϕ_2 as measured positively on the earth-side of the perihelion line. The inclination of the ray of light to the perihelion line on its arrival at the earth is

$$(\phi_2-\varpi)+\chi_2$$
$$=2\rho(\mathrm{K}-\mu_2)+\chi_2.$$

Hence the deflection of the ray, in passing from perihelion to the earth (measured in the same sense as before) is

$$2\rho(\mathrm{K}-\mu_2)+\chi_2-\tfrac{1}{2}\pi.$$

Consequently, the total deflection of the ray in passing from the star to the earth is

$$2\rho(\mathrm{K}-\mu_1)+\chi_1+2\rho(\mathrm{K}-\mu_2)+\chi_2-\pi.$$

The Approximate Expression for the Deflection of a Ray.

5. When we deal with approximations, we must select the orders. With the astronomical units in Professor Eddington's Report,

$$m = 1{\cdot}47 \text{ km.}$$

The sun's radius R is here taken to be

$$\mathrm{R} = 695{,}500 \text{ km.};$$

and judging from the position of the selected star in one of the diagrams contained in the memoir on the 1919 eclipse, we have (roughly)

$$\mathrm{D}=2\mathrm{R}.$$

Thus m/D may be taken as the small quantity of reference. Now

$$\frac{m}{\mathrm{R}}=2{\cdot}11\times10^{-6},\quad \frac{\mathrm{R}_2}{\mathrm{R}}=2{\cdot}15\times10^{2},\quad \frac{\mathrm{R}_1}{\mathrm{R}}=1{\cdot}63\times10^{9}.$$

Having regard to the magnitude of m/D, and to the degree of accuracy that is possible in observations, we omit $(m/D)^2$. We keep, where they occur, R/R_2 and its powers up to 10^{-9} inclusive, that is $\left(\frac{D}{R_2}\right)^4$; also we keep $\frac{m}{R_2}$, $\frac{mD}{R_2{}^2}$, $\frac{R}{R_1}$; the reason for the selection of the order will appear at the end of the calculations.

We have, up to the order retained,

$$\gamma = -\frac{1}{D} + \frac{2m}{D^2},$$

$$\alpha - \gamma = \frac{1}{2m} + \frac{1}{D},$$

$$\beta - \gamma = \frac{2}{D} - \frac{2m}{D^2},$$

and so

$$k^2 = \frac{4m}{D}.$$

Next,

$$\rho = 1 - \frac{m}{D},$$

$$K = \tfrac{1}{2}\pi\left(1 + \frac{m}{D}\right).$$

Beginning with the earth-sun portion of the path, we have

$$\gamma + (\beta - \gamma)\sin^2\theta_2 = \frac{1}{R_2},$$

so that

$$\cos 2\theta_2 = \frac{\frac{m}{D} - \frac{D}{R_2}}{1 - \frac{m}{D}}.$$

Now D/R_2 is greater than m/D; so, writing

$$2\theta_2 = \tfrac{1}{2}\pi + u,$$

we have

$$\sin u = \frac{\frac{D}{R_2} - \frac{m}{D}}{1 - \frac{m}{D}}$$

$$= \frac{D}{R_2} - m\left(\frac{1}{D} - \frac{1}{R_2}\right)$$

to our order; and therefore, also to the order,

$$u = \frac{D}{R_2} - m\left(\frac{1}{D} - \frac{1}{R_2}\right) + \tfrac{1}{6}\frac{D^3}{R_2{}^3}.$$

Thus

$$\theta_2 = \tfrac{1}{4}\pi + \frac{D}{2R_2} - \frac{m}{2D} + \frac{m}{2R_2} + \tfrac{1}{12}\frac{D^3}{R_2{}^3}.$$

Next,

$$\mu_2 = \int_0^{\theta_2} (1 - k^2 \sin^2\theta)^{-\frac{1}{2}} d\theta$$
$$= \theta_2(1 + \tfrac{1}{4}k^2) - \tfrac{1}{8}k^2 \sin 2\theta_2.$$

Now

$$\sin 2\theta_2 = \cos u$$
$$= 1 - \tfrac{1}{2}u^2;$$

the term arising out of $-\frac{1}{2}u^2$ in $-\frac{1}{8}k^2 \sin 2\theta_2$ is

$$\tfrac{1}{4}\frac{m}{D}\frac{D^2}{R_2^2},$$

which is of the order 10^{-11}, and is rejected under the selection made; hence

$$\mu_2 = \theta_2(1 + \tfrac{1}{4}k^2) - \tfrac{1}{8}k^2$$
$$= \theta_2\left(1 + \frac{m}{D}\right) - \frac{m}{2D}$$
$$= \tfrac{1}{4}\pi - \frac{m}{D}(1 - \tfrac{1}{4}\pi) + \frac{D}{2R_2} + \frac{m}{R_2} + \tfrac{1}{12}\frac{D^3}{R_2^3}.$$

Again,

$$\tan\chi_2 = \frac{R_2}{\left\{\frac{R_2^4}{D^3}(D - 2m) - R_2^2 + 2mR_2\right\}^{\frac{1}{2}}}$$
$$= \frac{D}{R_2} + \frac{m}{R_2} + \frac{D^3}{2R_2^3},$$

on neglecting terms $\frac{3}{8}\frac{D^5}{R_2^5}$ and $\frac{mD^2}{R_2^3}$ as being of too small an order; hence

$$\chi_2 = \tan\chi_2 - \tfrac{1}{3}\tan^3\chi_2 + \ldots$$
$$= \frac{D}{R_2} + \frac{m}{R_2} + \tfrac{1}{6}\frac{D^3}{R_2^3};$$

up to the order.

Consequently

$$2\rho(K - \mu_2) + \chi_2$$
$$= 2\left(1 - \frac{m}{D}\right)\left[\tfrac{1}{2}\pi\left(1 + \frac{m}{D}\right) - \tfrac{1}{4}\pi + \frac{m}{D}\left(1 - \tfrac{1}{4}\pi\right) - \frac{D}{2R_2} - \frac{m}{R_2} - \tfrac{1}{12}\frac{D^3}{R_2^3}\right]$$
$$+ \frac{D}{R_2} + \frac{m}{R_2} + \tfrac{1}{6}\frac{D^3}{R_2^3}$$
$$= \tfrac{1}{2}\pi + \frac{2m}{D},$$

accurately up to the order retained.

The calculations for the portion of the path between the star and perihelion are exactly similar. The order of approximation is much

closer because R_1 is so much larger than R_2. But the result is that, even to a higher order than before,

$$2\rho(K-\mu_1)+\chi_1=\tfrac{1}{2}\pi+\frac{2m}{D}.$$

Hence, finally the total deflection of the ray, as it moves from the star, passes the sun at a perihelion distance D and comes to the earth, being equal to

$$2\rho(K-\mu_1)+\chi_1+2\rho(K-\mu_2)+\chi_2-\pi,$$

is equal to

$$\frac{4m}{D},$$

accurately up to the order 10^{-9} at least among the quantities measured.

If approximation to a higher order is desired, so that it shall be accurate up to the order 10^{-12} at least, then the whole calculation must be revised. For then, m^2/D^2 becomes significant; and an added term mD^2/R_2^3, of the same order as m^2/D^2, is introduced into the deviation. But present powers of observation seem to forbid any practical test up to that order of accuracy.

The Approximate Equation of the Path.

6. The equation to the path, approximate to the same order, is obtained simply from the accurate equation. We have

$$u=\gamma+(\beta-\gamma)\frac{1+\mathrm{cn}\dfrac{\phi-\varpi}{\rho}}{1+\mathrm{dn}\dfrac{\phi-\varpi}{\rho}}.$$

Now, when k is small, as we have

$$k^{\frac{1}{2}}=\frac{2q^{\frac{1}{4}}+2q^{\frac{9}{4}}+\ \ldots}{1+2q+\ \ldots}$$

always, we have

$$\frac{k^2}{16}=\frac{q}{(1+2q)^4},$$

approximately, that is, up to our order,

$$q=\tfrac{1}{16}k^2.$$

Let

$$\phi-\varpi=\sigma,$$

$$x=\frac{\pi\sigma}{2\rho K};$$

then

$$\mathrm{cn}\frac{\sigma}{\rho}=(1-4q\sin^2 x)\cos x,$$

$$\mathrm{dn}\frac{\sigma}{\rho}=1-8q\sin^2 x,$$

approximately. Now

$$k^2 = 4\frac{m}{D},$$

so that

$$q = \frac{m}{4D};$$

also

$$K = \tfrac{1}{2}\pi\left(1 + \frac{m}{D}\right), \quad \rho = 1 - \frac{m}{D},$$

and therefore, to the order selected,

$$\frac{\pi}{2\rho K} = 1,$$

so that

$$x = \sigma = \phi - \varpi.$$

Further

$$\gamma = -\frac{1}{D} + \frac{2m}{D^2}, \qquad \beta - \gamma = \frac{2}{D}\left(1 - \frac{m}{D}\right);$$

hence

$$u = -\frac{1}{D} + \frac{2m}{D^2} + \frac{2}{D}\left(1 - \frac{m}{D}\right)\frac{1 + \cos x - \frac{m}{D}\sin^2 x \cos x}{2 - 2\frac{m}{D}\sin^2 x}$$

$$= \frac{1}{D}\cos x + \frac{m}{2D^2}(3 - 2\cos x - \cos 2x),$$

to our order: that is, the approximate equation of the path is

$$\frac{1}{r} = \frac{1}{D}\cos(\phi - \varpi) + \frac{m}{2D^2}\{3 - 2\cos(\phi - \varpi) - \cos 2(\phi - \varpi)\},$$

D being the apsidal distance.

The deviation of the path from the apsidal tangent is easily estimated by taking

$$\frac{\cos(\phi - \varpi)}{D} = \frac{1}{r_1},$$

which is the apsidal tangent; we have, at once,

$$\frac{1}{r} - \frac{1}{r_1} = \frac{m}{D^2}\left(1 - \frac{D}{r_1}\right)\left(2 + \frac{D}{r_1}\right).$$

Imperial College of Science and Technology, S. W.:
1921 *September* 12.

On the Motion of the Perihelion of Mercury. By W. M. Smart.

§ 1. One of the outstanding discrepancies of the Newtonian Law, namely, the advance of the Perihelion of Mercury's orbit at the rate of 43″ per century, has been satisfactorily accounted for by Einstein, all the efforts of dynamical astronomers having hitherto failed to provide a solution of the problem on the basis of the Newtonian Law in relation to the known bodies of the Solar system. On the assumption that the unaccountable portion of the perturbations on Mercury might be due to an undiscovered intra-Mercurial planet ("Vulcan"), Le Verrier calculated the mass, corresponding to different heliocentric distances, of the hypothetical planet (*Annales de l'Observatoire de Paris*, tome v.; Tisserand, vol. iv., chapter 29). Other hypotheses have been considered at various times—an inner ring of planetoids, the non-sphericity of the Sun, Hall's corrected "inverse square law," and so on. I consider here only the problem of the hypothetical planet.

Recently, doubt has been thrown on the validity of the interpretation of the Michelson-Morley experiment, and even on the experiment itself (*Memorie della Società Astronomica Italiana*, nuova serie, vol. i., No. 4). Whatever justification there may be for this assault on one of the principal foundations of the Theory of Relativity, I have considered it of interest to pursue the study of the effect of a hypothetical planet further, in relation to the advance of the Perihelion of Mercury, on the Newtonian Law. The present note may thus be regarded as an extension or completion of Le Verrier's calculations, the case here considered being that of the equilateral configuration in the three-body problem. It is established that the perturbations in this particular case are considerable. Moreover, no fewer than six Trojan planets are now known, all having the same mean motion as that of Jupiter, so that the equilateral configuration seems to be a definite feature of the Solar system.

The existence of the hypothetical Vulcan, interior to Mercury, engaged the speculations of astronomers for a long time, but Vulcan still remains undiscovered. As Le Verrier showed, the mass of Vulcan, sufficient to account for the advance of 43″ per century, ranges from $\frac{1}{6}$ to $2\frac{2}{3}$ times the mass of Mercury, for heliocentric distances of Vulcan ranging from 0·27 to 0·12, the heliocentric distance of Mercury being 0·39. The comparatively large masses found by Le Verrier indicate a considerable stellar magnitude for Vulcan which would almost certainly lead to the discovery of the planet during a total eclipse of the Sun. The large perturbations resulting in the equilateral three-body problem suggest the possible existence of a Vulcan of small mass at the same heliocentric distance as Mercury. The hypothetical body in this case will sometimes be referred to, later, as the "Trojan Vulcan." The problem, then, to be considered is: "To find the mass of the Trojan Vulcan so that the advance of the Perihelion of Mercury may be accounted for on the Newtonian Law, and to obtain a rough approximation to the maximum stellar magnitude of the hypothetical planet which would afford some indication of the possibility of its discovery (or not)." If the mass turns out to be a small fraction of that of Mercury, the

difficulty of establishing the existence of the planet observationally would be great: if the mass is comparable with that of Mercury, the first of Einstein's three crucial tests cannot be challenged.

§ 2. Let m denote the mass of the Trojan Vulcan and M the mass of Mercury. I assume for M the value $\frac{1}{6} \times 10^{-6}$ ⊙. It is certain that if a Trojan Vulcan exists, m cannot exceed M, for, otherwise, at maximum elongation the stellar magnitude would be of the same order as, or brighter than, that of Mercury under similar circumstances, and consequently the hypothetical planet could not possibly evade detection. The orders of the masses are therefore small, and the problem is similar in type to that discussed in "Librations of the Trojan Planets" (*Mem. R.A.S.*, 62).

We take the ecliptic as the plane of reference.

Let l denote the mean anomaly of Mercury.
g ,, the distance of Perihelion from the Node.
Ω ,, the longitude of the Node.
L ,, $a^{\frac{1}{2}}$.
G ,, $a^{\frac{1}{2}}(1-e^2)^{\frac{1}{2}}$.
H ,, $a^{\frac{1}{2}}(1-e^2)^{\frac{1}{2}}\cos i$.
r ,, the heliocentric distance of Mercury.

The corresponding elements of Vulcan are denoted by a suffix.

Let Δ denote the distance of Mercury from Vulcan
and S the angular ,, ,, ,,

Then, for Mercury, we have the canonic equations:—

$$\frac{dl}{dt} = -\frac{\partial f}{\partial \mathrm{L}} \quad : \quad \frac{d\mathrm{L}}{dt} = \frac{\partial f}{\partial l}$$

$$\frac{dg}{dt} = -\frac{\partial f}{\partial \mathrm{G}} \quad : \quad \frac{d\mathrm{G}}{dt} = \frac{\partial f}{\partial g}$$

$$\frac{d\Omega}{dt} = -\frac{\partial f}{\partial \mathrm{H}} \quad : \quad \frac{d\mathrm{H}}{dt} = \frac{\partial f}{\partial \Omega}$$

where

$$f = \frac{1}{2a} + m\left[\frac{1}{\Delta} - \frac{r\cos \mathrm{S}}{r_1^2}\right].$$

There is a similar set of equations for the Trojan Vulcan.

The canonic form is maintained if we make the following substitutions:—

$$l' = l + g + \Omega \quad : \quad \mathrm{L}' = \mathrm{L}$$
$$g' = \quad g + \Omega \quad : \quad \mathrm{G}' = -(\mathrm{L} - \mathrm{G})$$
$$h' = \quad \Omega \quad : \quad \mathrm{H}' = -(\mathrm{G} - \mathrm{H})$$

with similar equations for the hypothetical planet.

The longitude of the Perihelion of Mercury is thus g'.

The equations of motion become :—

$$\left.\begin{aligned} \frac{dl'}{dt} &= -\frac{\partial f}{\partial \mathrm{L}'} \quad : \quad \frac{d\mathrm{L}'}{dt} = \frac{\partial f}{\partial l'} \\ \frac{dg'}{dt} &= -\frac{\partial f}{\partial \mathrm{G}'} \quad : \quad \frac{d\mathrm{G}'}{dt} = \frac{\partial f}{\partial g'} \\ \frac{dh'}{dt} &= -\frac{\partial f}{\partial \mathrm{H}'} \quad : \quad \frac{d\mathrm{H}'}{dt} = \frac{\partial f}{\partial h'} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

with similar equations for Vulcan.

We require to consider only the terms of the disturbing functions which give rise to the important perturbations: these are the terms with argument $l' - l_1'$.

Let

$$\phi = l' - l_1'$$

and let F, F_1 denote the disturbing functions when all harmonic terms, not having arguments ϕ, are omitted. When F, F_1 replace f, f_1 in equations (1), we have at once that G', H', G_1', H_1' are constants, which we denote by $-\frac{1}{2}\epsilon^2$, $-2\sigma^2$, $-\frac{1}{2}\epsilon_1^2$, $-2\sigma_1^2$ respectively.

The important equation for our purpose is

$$\frac{dg'}{dt} = 2\frac{\partial \mathrm{F}}{\partial(\epsilon^2)}.$$

By omitting all but the principal terms, we reduce this equation to

$$\frac{dg'}{dt} = m\left[\frac{7}{16\,\mathrm{S}^3} - \frac{5}{16\,\mathrm{S}} + \cos\phi\right] \quad . \quad . \quad . \quad (2)$$

where

$$\mathrm{S} = \sin\frac{\phi}{2}.$$

The approximate value of $\frac{dg'}{dt}$ is given by putting

$$\phi = 60^\circ \quad : \quad \mathrm{S} = \tfrac{1}{2} \text{ in equation (2).}$$

Then

$$\frac{dg'}{dt} = \frac{27}{8}m \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We have to determine m from (3), so that $\frac{dg'}{dt}$ is 43″ per century.

In the preceding equations, the unit of time has been chosen so that the mass of the Sun is 1 and the mean heliocentric distance of the planets is 1. For Mercury, we have

$$a = 0{\cdot}387 \quad : \quad \text{Period} = 87{\cdot}97 \text{ days.}$$

From the formula

$$n^2a^3 = k^2(1+m)$$

the unit of time is found to be 14 days.

Now $\frac{dg'}{dt} = 43''$ per century. When this is expressed in circular measure and in terms of the unit of time, it becomes

$$\frac{dg'}{dt} = \frac{43 \times 14}{206265 \times 100 \times 365}.$$

The equation for m is therefore

$$m = \frac{8 \times 43 \times 14}{27 \times 7{\cdot}5 \times 10^9},$$

from which

$$m = \frac{1}{4{\cdot}2} \times 10^{-7};$$

that is, the ratio of the mass of Vulcan to that of Mercury is $\frac{1}{7}$.

§ 3. Since the mean heliocentric distances of Mercury and the hypothetical planet are the same, the effective light area of the planet at maximum elongation bears the same ratio to the total surface area as the corresponding quantities in the case of Mercury: and similarly, for any definite phase. The average maximum stellar magnitude of Mercury is about $-1{\cdot}6$ (*Nautical Almanac*): if the hypothetical planet has the same mean density and albedo as Mercury, its mean maximum stellar magnitude would be about 0·0. The density of Mercury is not known very accurately—it is probably about 5·0 (Newcomb)—and a density of 7·5 for the Trojan Vulcan may not be unreasonable as an upper limit. According to Zöllner (Young, *General Astronomy*), the albedos of the planets are:—

Mercury 0·13	Venus 0·50	Mars 0·26		
Jupiter 0·62	Saturn 0·52	Uranus 0·64	Neptune 0·46	
and Moon 0·16.				

There does not appear to be a definite correlation between the albedo of a planet and its mass—a not surprising result when the variability of the constituents and conditions of the light-reflecting surfaces are taken into consideration,—but there does appear to be a rather general correlation, so that for a small planet one would expect a small value for its albedo. Now, Vulcan's mass $= \frac{1}{7}$ of the mass of Mercury $= \frac{2}{3}$ mass of Moon approximately: as the extreme lower limit of the albedo of the hypothetical planet, we may take the value to be $\frac{1}{2}$ albedo of Mercury. With the extreme density 7·5, and extreme albedo 0·065, the mean maximum stellar magnitude of the planet would be approximately 1·2. Thus the mean maximum stellar magnitude of the hypothetical planet may be expected to lie between 0·0 and 1·2.

§ 4. At the total eclipse of Sun, 1878 July 29, a search was made for Vulcan in a systematical manner by Professor A. Hall and Professor S. Newcomb (*Observations, U.S. Naval Observatory*, 25; Preface, xi, xiii) with unsuccessful results. Sweeps were made up to 10° east and west of the Sun. At maximum elongation the angular distance of Mercury from the Sun is about 23°. As Vulcan was supposed to be a planet

well within the orbit of Mercury, the maximum elongation of the hypothetical planet was estimated to give an angular distance from the Sun of the order of 10°–12°. But if the hypothetical planet is a "Trojan," it is clear that its maximum distance from the Sun would be 23°, and accordingly its position in the sky at the time of the eclipse might be well outside the area swept.

On the assumption that the Trojan Vulcan precedes Mercury, I have worked out roughly the angular distance of the hypothetical planet from the Sun in two cases:—

(*a*) Heliocentric longitude of Vulcan = Heliocentric longitude of Mercury + 60°.
(*b*) Heliocentric longitude of Vulcan = Heliocentric longitude of Mercury + 70°.

From the *Nautical Almanac*, 1878, we have—

Heliocentric longitude of Mercury = 215°.
Longitude of the Sun = 126°.

The results are:—

for (*a*), angular distance from the Sun = 16°
for (*b*), ,, ,, ,, ,, = 11°.

It is clear that,

if the Heliocentric longitude of Vulcan = Heliocentric longitude of Mercury + (60° − α),

where α is a positive angle representing the libration in longitude, then the angular distance will exceed 16°. If we take 10° to represent approximately the maximum libration of Vulcan in longitude (an ample allowance when the smallness of the mass of Mercury is taken into account), the angular distance of the planet from the Sun would consequently be greater than 11°, and thus the Trojan Vulcan would be well outside the area swept.

On the assumption that Vulcan is a following planet, its position in the sky at the time of the eclipse, I find, would be well within the area swept, and consequently its detection would have been unavoidable.

It is of interest to note Hall's report: "The darkness during totality was very slight, and I could read my chronometer . . . with ease, and without the aid of artificial light" (Biographical Memoir of A. Hall, by G. W. Hill, *National Academy of Sciences' Publications*, 1908 April 23).

§ 5. The number of Transits of Mercury during the nineteenth century was 19: for a Trojan Vulcan with an inclination of the same order as that of Mercury, the number of transits would be about the same: with an inclination to the ecliptic considerably greater than that of Mercury, the number of transits would be very much smaller. However, the probability of detecting the hypothetical planet by this means, at observatories where the Sun's surface is studied with fair continuity, would appear to be considerable.

§ 6. The formula for the rate of advance of the Perihelion has been derived by omitting all but the principal terms. Among the rejected terms are harmonic terms of the form

$$A_s \cos s\theta_0(t+c) + B_s \sin s\theta_0(t+c),$$

in which θ_0 is the perturbation period. Unless this period is very large, the effect of these periodic terms ought to make itself apparent in the observed values of $\frac{dg'}{dt}$.

To find the period, let us write

$$a = 1 + \delta a \ : \ a_1 = 1 + \delta a_1 \ : \ \phi = 60^\circ + \delta\phi.$$

Then equations (1), F and F_1 being written for f and f_1, yield the set :—

$$\frac{d}{dt}(\delta a) = \tfrac{9}{2} m\delta\phi + \frac{3\sqrt{3}}{2} m(\delta a - \delta a_1)$$

$$\frac{d}{dt}(\delta a_1) = -\tfrac{9}{2} M\delta\phi + \frac{3\sqrt{3}}{2} M(\delta a - \delta a_1)$$

$$\frac{d}{dt}(\delta\phi) = -\tfrac{3}{2}(\delta a - \delta a_1) + 2(m - M) - \frac{3\sqrt{3}}{2}(m - M)\delta\phi + \tfrac{3}{2}(m\delta a - M\delta a_1) + 4(M\delta a - m\delta a_1).$$

If δa, δa_1, $\delta\phi$ are periodic functions of period $\frac{2\pi}{\theta_0}$, we derive easily the equation for θ_0, namely :—

$$\theta_0^3 - \tfrac{27}{4}\theta_0\{m + M - 2m^2 - \tfrac{10}{3} mM - 2M^2\} - \frac{135\sqrt{3}}{4} mM(m - M) = 0.$$

Having regard to the small values of m and M, we have

$$\theta_0 = \frac{3\sqrt{3}}{2}(m + M)^{\frac{1}{2}}.$$

The perturbation period is thus

$$\frac{4\pi}{3\sqrt{3}(m + M)}.$$

The value is found to be about 220 years.

It would seem that the effect of the small periodic terms (the period being of the order of 220 years) would be scarcely large enough to be detected during the last century by observation. For this reason, I have considered it unnecessary to investigate further the effect of these terms.

The equation which gives the perturbations in the longitude of the Node is

$$\frac{dh'}{dt} = \tfrac{1}{2}\frac{\partial F}{\partial(\sigma^2)}.$$

If we omit terms of the same order as we omitted in the case of $\frac{dg'}{dt}$ we have simply

$$\frac{dh'}{dt} = 0.$$

Thus the effect of Vulcan can introduce only second or higher order terms into the motion of the Node of Mercury's orbit, which, in comparison with the Perihelion effect, can only be negligible.

§ 7. There remains the consideration of the perturbations of the hypothetical planet on Venus and the earth.

The ratio a of the semi-major axes of Vulcan (or Mercury) and Venus is $\frac{19}{36}$ or $\frac{1}{2}$ approximately. The mean daily motion of Venus is $5''{\cdot}7 \times 10^3$, and the mean yearly motion n' may be taken to be $2'' \times 10^6$.

If g'' is the longitude of Perihelion of Venus and the unit of time is a year, we have

$$\frac{dg''}{dt} = Bmn',$$

where

$$B = \frac{3}{2}\left(\frac{a^2}{1-a^2}\right) + \frac{2{\cdot}4a^4}{(1-a^2)^2}.$$

Hence, for $a = \frac{1}{2}$,

$$B = \tfrac{1}{2} \quad \text{with sufficient accuracy.}$$

Consequently,

$$\frac{dg''}{dt} = \tfrac{1}{2} \times \tfrac{1}{4}\tfrac{0}{2}^{-6} \times 2 \times 10^6$$

$$= \tfrac{1}{42}'' \text{ per year.}$$

The rate of advance of the Perihelion of Venus thus comes out to be about $2''{\cdot}5$ per century.

This is of the order of the errors of observation in the case of the nearly circular orbit of Venus.

For the earth, the corresponding quantity would be negligible. The perturbations of the other elements of Venus and the earth come out equally small, and well within the errors of observation.

Summary.

§ 8. The present investigation may be regarded as the completion of Le Verrier's calculations for a hypothetical planet at the same mean distance from the Sun as Mercury. The mass of a disturbing planet, necessary to account for the well-known discrepancy, has been found to be $\frac{1}{7}$ mass of Mercury, and the maximum stellar magnitude has been estimated to be within the range 0·0 to 1·2. The existence of Trojan planets in the Solar system has been known since 1910, the number of

such planets now known being 6. Vulcan has been searched for at total Solar eclipses, and continuous observation of the Sun has failed to detect the transit of the hypothetical planet across the Sun's disc. The probability of the planet's existence seems from the foregoing to be very small. If the mass of the Trojan Vulcan had come out at $\frac{1}{700}$ or even $\frac{1}{70}$ mass of Mercury, the circumstances of the planet's non-discovery would have been easily explicable on observational grounds. For a Vulcan preceding Mercury, the Solar eclipse of 1878 would not have proved a favourable occasion for the detection of the planet, when the area swept is taken into consideration, although a star of magnitude 0·0 to 1·2 at a distance from the Sun of 12°–23° might have been expected to attract attention. I have not found any other eclipse records in which the search for Vulcan was so systematically carried out.

Observatory, Cambridge:
1921 *Nov.* 10.

Star-streaming in the Region round the North Pole.

By Sir Frank Dyson. (Plate 1.)

The recently published vol. iv. of the Greenwich Section of the Astrographic Catalogue gives the proper motions of a large number of stars between 65° N. dec. and the Pole. These are obtained by comparison of recent observations with those of the *Astronomische Gesellschaft* and earlier catalogues, supplemented by the direct comparison of photographs from fifteen to twenty years apart. All stars contained in the *Bonn Durchmusterung* between 65° N. dec. and the North Pole have been examined, and probably most of these stars which have considerable proper motion have been detected. There are

257 stars with p.m. $>20''$ a century
239 ,, ,, ,, between $15''$ and $20''$ a century
565 ,, ,, ,, ,, $10''$ and $15''$ a century.

The area is ·047 of the whole sky, so that in the whole sky there are about 10,000 stars with p.m. $>15''$ a century to this limit of magnitude, say $10^{m}\cdot3$ on the Harvard scale—a figure which may be compared with 40,000 given by Dr. Max Wolf* going to a magnitude of 15 approximately. Within the B.D. limit of magnitude he finds rather less than one-fourth of this number.

If the direction of motion is projected on the tangent plane at the Pole, the distribution of the directions is as shown in the following table, where the direction from the Pole to the first point of Aries is called 0° and to a point 6^{h} is called 90°.

* *Veröffentlichungen der Badischen Sternwarte zu Heidelberg*, Band 7, No. 10.

Distribution of Direction of Proper Motions of Stars within 25° of the North Pole.

Dir.	No.	Dir.	No.	Dir.	No.	Dir.	No.
0–10	4	90–100	161	180–190	13	270–280	18
10–20	10	100–110	130	190–200	9	280–290	13
20–30	7	110–120	73	200–210	6	290–300	18
30–40	8	120–130	51	210–220	10	300–310	11
40–50	23	130–140	34	220–230	16	310–320	5
50–60	34	140–150	24	230–240	18	320–330	4
60–70	55	150–160	9	240–250	19	330–340	3
70–80	81	160–170	11	250–260	19	340–350	3
80–90	129	170–180	10	260–270	14	350–0	8

From these figures and the accompanying diagram it is seen that the stars moving in directions from 40° to 150° all belong to Stream I., whose apex is seen to be in the direction of 94° or 95°. This accounts for 795 of the 1061 stars. We may reasonably go a little further and include nearly all the 830 stars between 20° and 170° as belonging to Stream I.

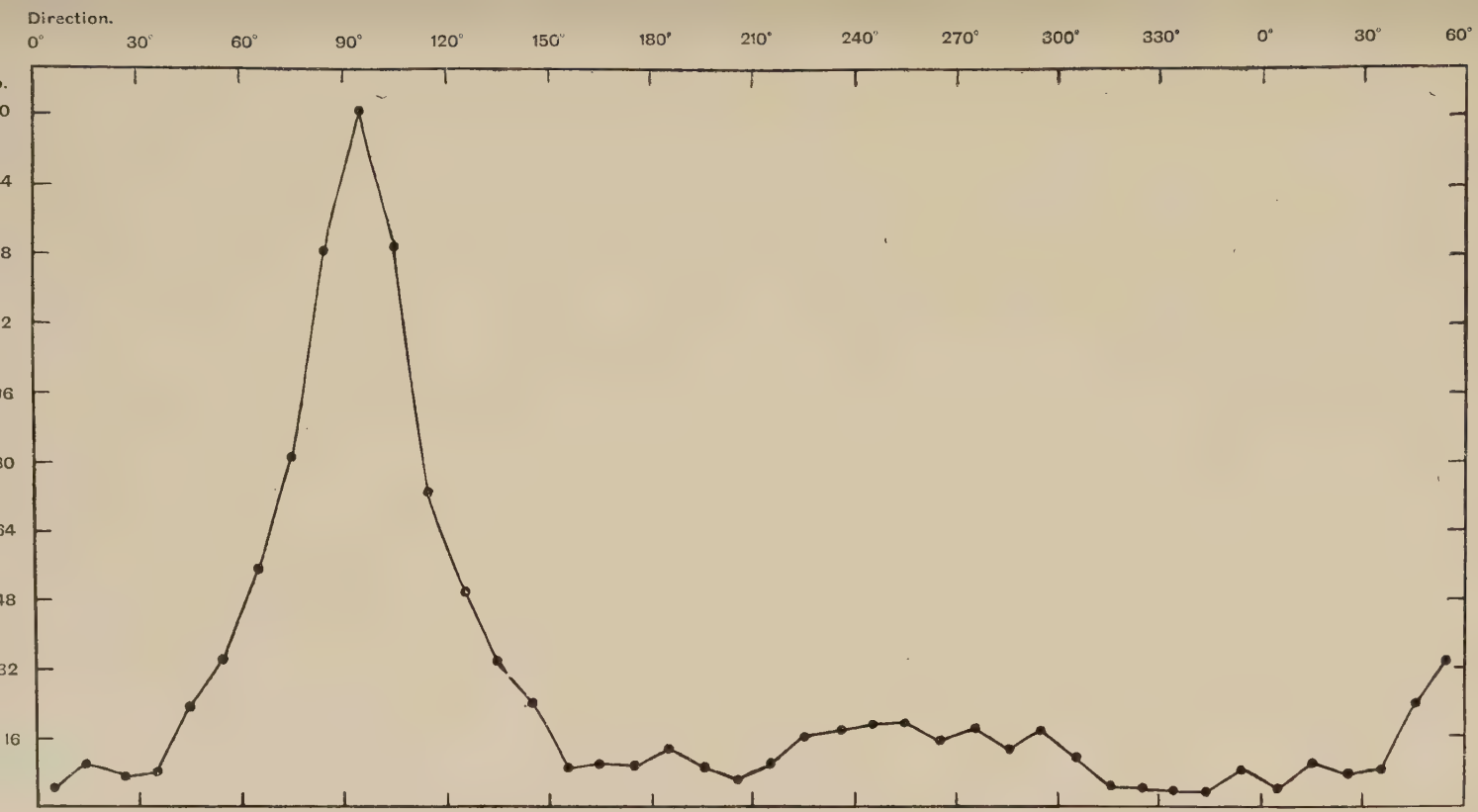
The stars moving in the direction 200° to 320° may be regarded as belonging to Stream II. The right ascension of the apex is seen to be at 260°. A graphical discussion showed that the declination of the apex was at about −60°. Assuming this to be the position of the apex of Stream II., the directions of the proper motion of each star which fell between the directions 170° to 20° (*i.e.* for stars not belonging to Stream I.) were compared with the direction from each star to the apex.

Of these 231 stars, 57 are within 30° of the antapex and have been excluded. For the remaining 174, the divergences from the direction of their proper motions from that of the apex of Stream II. are as follows:—

	No. of Stars.	
	+	−
Between 0° and 10°	16	13
10 ,, 20	13	17
20 ,, 30	11	13
30 ,, 40	13	7
40 ,, 50	5	7
50 ,, 60	5	9
60 ,, 90	14	14
>90	4	12

The stars moving in directions near 180° were examined in detail; they do not indicate any special radiant. The figures and diagrams show to what a great extent "two star streams" sums up the motion of these stars.

Royal Observatory, Greenwich:
1921 Nov. 7.



DISTRIBUTION OF DIRECTION OF PROPER MOTIONS.

A Determination of the Photographic Magnitude Scale of the North Polar Sequence. By H. Spencer Jones, M.A., B.Sc.

(*Communicated by the Astronomer Royal.*)

1. The North Polar Sequence consists of a series of stars, graded in magnitude, in the neighbourhood of the North Pole, which were selected by the late Professor E. C. Pickering to serve as a basis for the determination, by purely comparative methods, of the magnitudes of stars in other areas. The stars of the primary sequence were chosen as far as possible from amongst the stars whose spectra did not differ greatly from type Ao. This sequence was supplemented by another sequence, in which this condition was not adhered to, and also by a series of red stars of late spectral type.

Extensive investigations of the magnitudes of some or all of the stars of the combined sequence have been made at Harvard, Mount Wilson, Greenwich, Potsdam, and elsewhere. These investigations have served to confirm the photographic magnitudes originally determined at Harvard between the limits of magnitude 10 and 15. For both the brighter and fainter stars, however, systematic differences have been obtained. With the faint stars we are not concerned here. A determination of the magnitudes of the brightest stars of the sequence made by Seares at Mount Wilson with the 60-inch reflector showed discordances from the Harvard magnitudes amounting for some stars to as much as four-tenths of a magnitude. These bright stars were not included within the range of the Greenwich and Potsdam investigations, and have been fully covered only at Harvard and Mount Wilson. Further investigation of their magnitudes was greatly to be desired, in view of the importance of an accurate scale of magnitudes for statistical investigations and for other purposes. Professor Seares suggested that a determination of the magnitudes of the sequence stars brighter than 10^m, with a view to checking the Harvard and Mount Wilson scales, should be made the subject of an independent investigation. The investigation, the results of which are given in the present paper, covers all the stars of the North Polar Sequence, including the supplementary and red stars, down to a magnitude limit of about 13^m.

2. The telescope used for obtaining the photographs was the Astrographic Equatorial, of 13 inches aperture and 135 inches focal length. The observations were made by Messrs. Jones, Melotte, and Barton. The telescope was employed in conjunction with a parallel wire diffraction grating placed in front of the objective. The magnitude interval between the principal and first diffracted images obtained with this grating was determined theoretically from measurements of the mean diameter of the wire (d) and of the total grating interval ($a+d$), using the formula

$$\text{Magnitude interval} = 5\,(\log a\pi/a+d - \log \sin a\pi/a+d).$$

The determinations of d and of $a+d$, made on two separate occa-

sions by Messrs. Jones and Melotte, from a mean of measures made at a large number of points of the grating, were as follows:—

Date.	Measurer.	d. in.	$a+d$. in.	Mag. Interval. m
1920 Aug. 12	H.J.	·0634	·2770	2·831
,,	P.M.	·0631	·2769	2·841
1921 Jan. 3	H.J.	·0635	·2770	2·830
,,	P.M.	·0632	·2766	2·836

The value $2^m{\cdot}83$ was adopted throughout the investigation. The first diffracted images given by this grating are round and sharp, except when the image is near the limit of visibility. In such cases, the diffracted images were not measured. For bright stars, the second diffracted images may sometimes be used with advantage to supplement the information derived from the first diffracted image. The second image is slightly elongated, but its estimation presents no difficulties in the case of a bright star near the centre of the plate. The ratio of the brightness of the second to that of the first diffracted image is $\cos^2 a\pi/a+d$, corresponding to a magnitude difference of $-5\log\cos a\pi/a+d$. With the grating used in this investigation, this difference is $0^m{\cdot}62$, so that the difference in magnitude between the central and second diffracted images is $3^m{\cdot}45$. Direct comparison between first and second diffracted images gives a difference of $0^m{\cdot}65$, corresponding to a magnitude difference between central and second diffracted images of $3^m{\cdot}48$. The difference is probably due to a slight systematic error of measurement in the case of the elongated images. The value $3^m{\cdot}48$ has been used in the reductions.

3. The measurement of the plates consisted in estimating the sizes of the star images by comparison with a graded scale of images, photographed for the purpose with the astrographic telescope. The average magnitude difference between successive images of this scale is about $0^m{\cdot}55$. The micrometer employed was one which had been previously used at Greenwich in photometric investigations, but early in the course of the present investigation it was reconstructed, with a view to rendering the comparison between a field image and the scale images easier and more accurate. It consists essentially of a double microscope, the plate being viewed through one objective and the comparison scale through the other. A prism system is utilised to bring both objects into focus in the field of view of the eyepiece. In the reconstruction, the centre prism combination of a Barr and Stroud rangefinder was used for this purpose: with this combination, the field image and the scale images can be brought into close proximity on either side of the dividing line, rendering the estimation quick and accurate. The plate holder can be moved in two perpendicular directions, so that any desired star can be brought into the centre of the field of the eyepiece: the scale is movable by a fine screw, by means of which the images can be moved parallel to the dividing line in the field.

Each plate was measured by two measurers, one measuring the right-hand images in the micrometer and the other the left-hand. The measures were made throughout by Messrs. Jones and Jeffries.

4. The scale comprised 16 images numbered (commencing with the brightest) 1 to 16. Only the portion from images 7 to 16 was utilised, however, as sufficient material was not available for the calibration of the bright portion of the scale. For the portion of the scale that was used, preliminary corrections to reduce the successive scale intervals to a uniform sequence were first derived by measuring the diameters of the images in a filar micrometer and plotting the square roots of these values against a uniform scale. Assuming that a linear relation should result, corrections to the various scale readings, in decimals of a scale interval, were deduced. This assumption implies that a relationship of the form $m = a + b\sqrt{d}$ holds over the magnitude range involved. This is not strictly accurate. Supplementary corrections were therefore deduced from the measures of all the Pole plates: for this purpose the Mount Wilson magnitudes for the stars of the Polar Sequence fainter than 10^m were provisionally assumed and these magnitudes were plotted against the estimated scale readings, to which the provisional corrections had first been applied. A straight line was then drawn to represent the results as accurately as possible, and the residuals were read off. The exposures on the Pole Field ranged from a few minutes to half an hour, and were sufficiently varied to smooth out accidental errors in the assumed magnitudes. Collecting the residuals for each scale interval from all the plates, supplementary corrections were deduced, which were combined with the provisional values and used as definitive corrections throughout. The relationship between magnitude and corrected scale estimate for any plate should therefore be linear within the range of the scale employed. After plotting the measures of a plate, a straight line was drawn to represent all the results as accurately as possible.

No bias towards the Mount Wilson scale is introduced by the above procedure, since between magnitudes 10^m and 15^m the Mount Wilson and Harvard magnitudes are in essential agreement.

5. The plates exposed on the Pole Field alone were first measured. These comprised eleven plates, with the Pole at the centre and exposures ranging from 13^m to 30^m; four plates, with N.P.S. 11 at the centre and exposures of 10^m; four plates, with N.P.S. 12 at the centre and exposures of 15^m, and one plate, with N.P.S. 13 at the centre, exposure 15^m. The estimated scale readings of all the sequence stars fainter than 10^m, corrected as described in the preceding paragraph, were plotted against the Mount Wilson magnitudes: for each plate a straight line was drawn to represent the plotted points, and the magnitude of every star measured on the plate and of the diffracted images of the brighter stars was read off. The portion of each plate measured was limited in each co-ordinate by the réseau lines 6 and 22, the centre of the plate being at the réseau reading 13·5 in each co-ordinate, and the value of one réseau interval being 5 minutes of arc. Within this area the images given by the astrographic telescope are sensibly round and suitable for measurement, and the distance correction is negligible.

From the measures of the brighter stars, for which the diffracted images were not fainter than scale-image 15, the measured value of the grating interval was determined. Images fainter than scale image 15 are near the limit of the plate and cannot be estimated with very great

accuracy in terms of the scale: they were therefore discarded. From these twenty pole plates in this way was deduced a grating interval of $2^m{\cdot}81$, which compares with a calculated interval of $2^m{\cdot}83$. This agreement may be accepted as a justification of the use of the calculated interval. Alternatively, accepting the calculated value as necessarily correct, the agreement with the measurements confirms the light-ratio of the Polar Sequence between the range of 10^m and $13^m{\cdot}5$. The correctness of the light-ratio between 10^m and 15^m has been confirmed, as already mentioned, by various other researches. A subsequent determination of the grating interval, using the material available from all the plates, gave the value $2^m{\cdot}82$.

From the measurement of the exposures on the Pole Field, corrections to the assumed magnitudes of the sequence stars fainter than 10^m were deduced, and the corrected values were employed in the reductions of the measurement of plates which were exposed on other fields.

The exposures on the Pole Fields served also to determine the magnitudes of the stars N.P.S. 10 (B.D. $+89°$ 3) and 11 (B.D. $+89°$ 29), which are the two brightest stars within about 30′ of the Pole, having magnitudes between 9^m and 10^m. Their magnitudes were deduced from the magnitudes of the diffracted images, using the calculated grating interval of $2^m{\cdot}83$. For N.P.S. 11 the magnitudes of the principal images were also read off from the plotted lines, a very slight extrapolation being involved.

6. For the brighter stars of the sequence (with the exception of the few stars brighter than $5^m{\cdot}5$), magnitudes were deduced by comparing the star with the Pole Field. An exposure was first given on the Pole; then two equal exposures of the same length as the Pole exposure were given on the sequence star in question, and finally a second exposure on the Pole. The length of the exposure was so chosen as to give a conveniently measurable diffracted image of the sequence star. From the measures of the Pole Field, the straight line connecting the scale estimations with magnitude for the particular plate under investigation was constructed; this enabled the magnitude of the first diffracted image of the central star to be deduced; in some cases, when the second diffracted image was sufficiently well defined, its magnitude was also read off and used to supplement the magnitude-value deducible from the first diffracted image.

Occasionally two bright sequence stars are sufficiently close together to make it worth while measuring either on plates taken with the other at the centre. Examples of such pairs of stars are N.P.S. 1 and 4 (B.D. 86° 269 and 272); N.P.S. 7 and 2*s* (B.D. 88° 74 and 71); N.P.S. 8 and 9 (B.D. 88° 9 and 13); N.P.S. 2*r* and 4*r* (B.D. 88° 112 and 114); and N.P.S. 3*r* and 5*r* (B.D. 88° 76 and 89° 22). Additional material is thus obtained for deriving the magnitudes of many of the brighter stars.

A minimum of six good plates for each star was obtained; plates which on measurement were found to have unsatisfactory images, due to poor definition or errors in driving, or in which the Pole images were unequal, were discarded.

The magnitudes of the stars in the Pole Field, used for comparison, range from a bright limit of $9^m{\cdot}1$. The magnitude of the comparison

sequence star can therefore be determined, without extrapolation, from the magnitude of the first diffracted image if the star is not brighter than $6^{m}\cdot3$, and from the second diffracted image if it is not brighter than $5^{m}\cdot7$.

For a few of the bright stars further material at the brighter end was obtained by photographing a sequence star of intermediate brightness near the centre of the plate. The exposures on the Pole Field were then given first and last, with two intermediate exposures on each of the two sequence stars. Thus, for instance, on some of the plates taken for N.P.S. 3 (mag. $5^{m}\cdot8$), two exposures were also given with N.P.S. 7 (mag. $7^{m}\cdot3$) near the centre. From the magnitude of the diffracted image of N.P.S. 7, derived from comparison with the Pole Field, the magnitude of the principal image is obtained. This image is also measured and can then be used to continue the plot to a magnitude much brighter than that of the first diffracted image of N.P.S. 3. In this way, also, N.P.S. 4 was compared on some plates with N.P.S. 7 and on others with N.P.S. 8 and 9; N.P.S. 5 was compared with N.P.S. 8 and 9, etc.

7. The magnitudes of the three brightest stars in the sequence—Polaris, δ Urs. Min., and 32 H Cephei—cannot be determined by direct comparison with the Pole Field using the grating employed in the present investigation. Their magnitudes may be determined, however, in a convenient manner by direct comparison with some of the stars of intermediate brightness. In order to secure as much material as possible for the plot connecting magnitude and scale estimation for any particular plate, advantage was taken of the groupings of certain of the bright stars. Polaris with stars N.P.S. 5, 8, and 9 form a group, and for the plates secured for the determination of the magnitude of Polaris the telescope was set on the centre of this group, so that these four stars are obtained for the same exposure and all within a short distance of the centre. On a number of these plates sequence stars 1 (δ Urs. Min.) and 4 were also photographed, the telescope being set for their mean R.A. and Dec. The magnitudes of sequence stars 4, 5, 8, and 9 having previously been determined, six points are available for the plotting of the scale estimation-magnitude relation, by using both principal and diffracted images of the two brighter stars. If, in addition, second diffracted images are also used, eight points become available. This number is amply sufficient for a good determination of the relation. From the plot the magnitude of the first diffracted image of N.P.S. 1 may be obtained, and that of the principal image used to extend the range beyond the point required for deducing the magnitude of Polaris. Plates with two exposures on the group N.P.S. 1*s*, 5, 8, and 9, and two on the pair N.P.S. 1 and 4, serve therefore to determine the magnitudes of both N.P.S. 1*s* (Polaris) and 1 (δ Urs. Min.).

For the determination of the magnitude of N.P.S. 2 (32 H Cephei) two exposures were given on the group N.P.S. 1*s*, 5, 8, and 9, two exposures on the pair N.P.S. 2*s* and 7, the telescope being set for their mean R.A. and Dec. and two exposures on N.P.S. 2. The length of each exposure being only one minute, the time occupied in obtaining a single plate was less than 10 minutes. These plates give the magnitude both of N.P.S. 2 and of N.P.S. 1*s*.

8. The deviations of individual points from the straight line representing as accurately as possible the measures of any one plate may be due to accidental error in estimation, to an error in the assumed magnitude, or to the use of incorrect scale corrections. The accidental error in estimation varies with the quality of the night: the scale used was obtained on a night of good definition and the images obtained under conditions of inferior definition are not exactly comparable with the scale images. To avoid, as far as possible, errors arising from this cause, observations were restricted to nights of reasonably good definition. With a sufficient number of plates the accidental error of measurement should be eliminated. With a good plate, with equal Pole images, the differences between the estimations of the two images of one star rarely exceed three-tenths of a scale interval, representing in magnitude about $0^m{\cdot}15$. The scale corrections used are believed to be accurate within very narrow limits; after their definite adoption, the accumulation of material from subsequent measurements did not indicate that the adopted values required any further revision. The mean deviation obtained from a large number of plates for any individual star may therefore be used as a correction to the assumed magnitude. Thus the plates taken primarily for the magnitudes of the brightest stars were utilised also for the revision of the magnitudes assumed for the intermediate stars, N.P.S. 4, 5, 7, 8, 9, and 28. In particular, the series of plates used for the determination of the magnitude of N.P.S. 2 give an accurate correlation of the magnitudes of N.P.S. 5 and 28. These two stars are nearly equal in photographic magnitude: the difference in the scale estimations for the two stars, converted into magnitude, gives a value of their magnitude difference almost independent of any possible errors in scale corrections or in the magnitudes assumed for other stars. Direct comparison was possible between their principal images, their first diffracted and on some plates their second diffracted images also.

It is hoped that in this way a homogeneous self-consistent series of magnitudes has been obtained.

In the reduction of the results, a correction has been applied to allow for differential atmospheric extinction whenever the fields compared on the same plate were not photographed at the same altitude. Had the endeavour to photograph each field when at the altitude of the Pole been too rigorously adhered to, it would not have been possible to complete the programme of observation within the desired time. Moreover, for the brightest stars, the condition could not be fulfilled.

The correction applied was $0^m{\cdot}01$ per 1° difference in altitude. This value was used by Schwarzschild in the *Göttingen Aktinometrie*. The correction is small, and as in general some plates of any given star were obtained above Pole and others below Pole, the mean correction for many of the stars is negligible.

9. *Results.* (i) *Primary Sequence.*—The results obtained can only be given in summarised form, on account of limitations of space. For the brighter stars, the nature of the material is given, the number of plates, the mean magnitude, the extreme range from individual plates, and the mean discordance. In arriving at these results, the magnitude interval between principal and first diffracted image has been adopted

as $2^m.83$, and that between principal and second diffracted as $3^m.48$. Corrections for differential atmospheric extinction have been applied.

N.P.S. 1.—Fourteen plates obtained. Magnitude deduced by comparison with adopted magnitudes of N.P.S. 4, 5, 8, and 9. First and second diffracted images measured on each plate. Mean magnitudes deduced from first and second diffracted images respectively are $4^m.37$ and $4^m.39$. The range in the magnitudes deduced from individual plates is $0^m.15$ and the mean discordance $0^m.03$. The adopted magnitude is $4^m.38$.

N.P.S. 2.—Eleven plates obtained. Magnitude deduced by comparison with N.P.S. 2*s*, 5, 7, 8, and 9. First and second diffracted images measured on each plate: deduced mean magnitudes $5^m.23$ and $5^m.24$ respectively. The range in the magnitudes deduced from individual plates is $0^m.12$ and the mean discordance is $0^m.04$. The adopted magnitude is $5^m.24$.

N.P.S. 3.—Fifteen plates obtained. On seven of these, the star was compared directly with the Pole Field: on the remaining eight, N.P.S. 7 was also photographed at the centre as an intermediary star. The second diffracted image was measured on two of the former seven plates and on the latter eight. The mean magnitude deduced from the former series is $5^m.81$ and from the latter $5^m.74$. The mean value from fifteen first diffracted images is $5^m.79$, and from ten second diffracted images is $5^m.70$. The fifteen plates give a range of $0^m.36$, a mean discordance of $0^m.08$, and a final magnitude of $5^m.77$.

N.P.S. 4.—This star was compared directly with the Pole on eleven plates: on four of these plates N.P.S. 8 and 9 were also photographed at the centre. First and second diffracted images were measured on all these plates. The mean magnitudes given by them are respectively $5^m.87$ and $5^m.88$. The magnitude adopted from these plates is $5^m.87$; the range is $0^m.20$ and the mean discordance $0^m.03$.

The star was also obtained on fourteen plates used for determining the magnitude of N.P.S. 1. The magnitudes read from the scale-magnitude relations for these plates—using principal, first diffracted, and for four plates the second diffracted image—have a mean value of $5^m.86$, with a range of $0^m.19$ and a mean discordance of $0^m.04$.

The magnitude adopted for the star is $5^m.87$.

N.P.S. 5.—This star was obtained at the centre of eight plates and compared directly with the Pole and with N.P.S. 8 and 9. First and second diffracted images were measured on all these plates, giving mean magnitudes of $6^m.41$ and $6^m.42$ respectively. The mean value for these plates is $6^m.42$, with a range of $0^m.21$ and a mean discordance of $0^m.07$.

The star was measured also on four plates taken with N.P.S. 4, 8, and 9 at the centre and exposed also on the Pole. The images on these plates are about 35′ from the centre. First and second diffracted images were measured. The mean magnitude obtained is $6^m.59$, with a range of $0^m.06$.

The magnitudes of this star were also read off from the results obtained for the bright star plates. From the fourteen plates exposed for N.P.S. 1, a mean magnitude is deduced of $6^m.47$, with a range of $0^m.35$ and a mean discordance of $0^m.09$. From ten plates exposed for

N.P.S. 2, a mean magnitude is deduced of $6^{m}\cdot42$, with a range of $0^{m}\cdot21$ and a mean discordance of $0^{m}\cdot06$.

The first series of eight plates receive the greatest weight, and the adopted mean value for the whole series is $6^{m}\cdot45$.

But on the ten plates exposed for N.P.S. 2, a direct comparison is obtainable between N.P.S. 5 and 2*s*, which are equidistant from the centres of the plates and of nearly equal magnitudes. This comparison is almost independent of any errors in the assumed scale corrections or of any uncertainty in the representation of the results of any single plate by a straight line, and therefore receive great weight. From the mean of the comparisons of principal images on ten plates, corrected for differential extinction, the difference of magnitudes of N.P.S. 5 and 2*s* is $-^{m}\cdot06$; from comparisons of first diffracted images on ten plates is obtained $-^{m}\cdot07$; from comparisons of second diffracted images on six plates is obtained $-^{m}\cdot03$. The adopted difference of magnitude is $-^{m}\cdot06$. Combining this with the results obtained for N.P.S. 2*s*, the magnitude finally adopted for N.P.S. 5 is $6^{m}\cdot46$.

N.P.S. 6 was obtained on fourteen plates, being compared directly with the Pole. The first and second diffracted images were measured on fourteen and nine plates respectively, and gave mean magnitudes of $7^{m}\cdot08$ and $7^{m}\cdot13$ respectively. The mean value from all plates is $7^{m}\cdot10$, with a range of $0^{m}\cdot27$ and a mean discordance of $0^{m}\cdot06$.

N.P.S. 7.—This star was compared with the Pole and photographed at the centre of the plate on fifteen plates, on eight of which N.P.S. 3 was also photographed at the centre. First diffracted images were measured on all the plates and second diffracted images on six of the latter plates. The mean magnitude deduced from fifteen first diffracted images is $7^{m}\cdot30$, and from six second diffracted images $7^{m}\cdot37$. The individual plates give a mean value of $7^{m}\cdot32$, with a range of $0^{m}\cdot29$ and a mean discordance of $0^{m}\cdot07$.

The magnitude of the star may also be deduced from the six plates taken for the magnitude of N.P.S. 2*s*, with the latter star at the centre and compared with the Pole. First diffracted images only were measured. The mean magnitude so obtained is $7^{m}\cdot33$, with a range of $0^{m}\cdot32$ and a mean discordance of $0^{m}\cdot09$. On these plates N.P.S. 7 is at a distance of 35′ from the centre of the plate.

From the plates taken specially for N.P.S. 2 the magnitude of N.P.S. 7 may also be read off. The principal image only was available. The mean magnitude thus deduced from ten plates is $7^{m}\cdot35$, with a range of $0^{m}\cdot33$ and a mean discordance of $0^{m}\cdot07$.

N.P.S. 8 and 9.—These stars are close together and do not differ greatly in magnitude. Their magnitudes were therefore deduced from the same plates.

The pair of stars were photographed at the centre and compared directly with the Pole on six plates, N.P.S. 4 being also photographed at the centre of four of these plates. For N.P.S. 8 both principal and first diffracted images were measured, the magnitude of the principal image being obtained by a slight extrapolation. The mean magnitude deduced from these six plates is $8^{m}\cdot33$, with a range of $0^{m}\cdot27$ and a mean discordance of $0^{m}\cdot07$. For N.P.S. 9, the first diffracted image

was measured on one plate only. The mean magnitude from the six plates is $8^m\cdot95$, with a range of $0^m\cdot20$ and a mean discordance of $0^m\cdot07$.

These stars were photographed also with N.P.S. 5 on the eight plates, on which the latter star was obtained at the centre and compared with the Pole Field. On these plates, the two stars are about 40′ from the centre. For N.P.S. 8, measuring both principal and first diffracted images, the mean magnitude deduced is $8^m\cdot35$, with a range of $0^m\cdot10$ and a mean discordance of $0^m\cdot02$. For N.P.S. 9, measuring second diffracted images on two plates only and principal images on all, the mean magnitude deduced is $9^m\cdot03$, with a range of $0^m\cdot21$ and a mean discordance of $0^m\cdot07$.

The magnitudes were also read off from the plots for the bright star plates (taken for N.P.S. 1 and 2). The principal images only were shown on these plates. From the mean of twenty-five plates, the magnitude of N.P.S. 8 was found to be $8^m\cdot37$, with a range of $0^m\cdot35$ and a mean discordance of $0^m\cdot06$. N.P.S. 9 was measured on nineteen plates, which gave a mean magnitude of $9^m\cdot00$, with a range of $0^m\cdot38$ and a mean discordance of $0^m\cdot06$. The adopted magnitudes are—for N.P.S. 8, $8^m\cdot35$, and for N.P.S. 9, $8^m\cdot98$.

10. (ii) *Supplementary Sequence Stars. N.P.S. 1s* (Polaris). — Polaris was measurable on most of the plates obtained for the two bright stars N.P.S. 1 and 2. A common exposure was given on the group N.P.S. 1*s*, 5, 8, and 9, with the telescope set for a mean R.A. and Dec. Polaris is about 15′ from the centre of the plate. On a few plates, only the second diffracted image was measurable, the first diffracted image being too bright and merging into the principal image. From fourteen plates exposed for N.P.S. 1, the magnitude deduced from measurements of nine first diffracted images is $2^m\cdot56$, and from fourteen second diffracted images is $2^m\cdot49$. The combined results give a mean value of $2^m\cdot52$, with a range of $0^m\cdot23$ and a mean discordance of $0^m\cdot06$. After correction for the variation of Polaris the mean value becomes $2^m\cdot53$.

From the plates obtained for N.P.S. 2, the magnitude of Polaris is most accurately determined by direct comparison between the principal image of N.P.S. 2 and the first diffracted image of Polaris. From nine plates, the mean difference in magnitude is found to be $0^m\cdot11$, N.P.S. 2 being the brighter, with a range of $0^m\cdot16$ and a mean discordance of $0^m\cdot05$. This difference becomes $0^m\cdot13$, after correction for the variation of Polaris. The magnitude of N.P.S. 2 has previously been adopted as $5^m\cdot24$, whence it follows that the magnitude of Polaris (at mean brightness) is $2^m\cdot54$.

The value $2^m\cdot54$ is adopted for the mean magnitude.

N.P.S. 2s.—This star was photographed at the centre of the plate and compared directly with the Pole on six plates. First diffracted images only were measured. The mean magnitude found from these plates is $6^m\cdot49$, with a range of $0^m\cdot27$ and a mean discordance of $0^m\cdot06$.

The star was also measured on fifteen plates taken for N.P.S. 7, this star being at the centre and N.P.S. 2*s* about 35′ away. Second diffracted images were measured on eight of these plates. The mean magnitude derived from the fifteen first diffracted images is $6^m\cdot62$, and from the eight second diffracted images is $6^m\cdot55$. The mean magnitude from

the fifteen plates is $6^m{\cdot}60$, with a range of $0^m{\cdot}34$ and a mean discordance of $0^m{\cdot}08$.

From the nine plates obtained for N.P.S. 2, it was deduced—as explained under N.P.S. 5—that the difference of the magnitudes of N.P.S. 5 and 2*s* is $0^m{\cdot}06$. These plates also give a mean magnitude for N.P.S. 2*s* of $6^m{\cdot}48$.

From the whole of this material, a final mean magnitude of $6^m{\cdot}52$ is deduced.

N.P.S. 3*s*.—The magnitude of this star is deduced from nine plates, on each of which it was photographed at the centre and compared with the Pole. First and second diffracted images were measured on each plate, giving mean magnitudes respectively of $6^m{\cdot}66$ and $6^m{\cdot}64$. The adopted mean is $6^m{\cdot}65$, with a range of $0^m{\cdot}18$ and a mean discordance of $0^m{\cdot}04$.

11. (iii) *Red Stars. N.P.S.* 1*r*.—Eleven plates were obtained of this star, which was photographed at the centre of the plate and compared directly with the Pole. First and second diffracted images were measured respectively on eleven and six plates, and gave mean magnitudes of $6^m{\cdot}73$ and $6^m{\cdot}70$. The magnitude adopted from the eleven plates is $6^m{\cdot}72$, with a range of $0^m{\cdot}44$ and a mean discordance of $0^m{\cdot}09$.

N.P.S. 2*r* and 4*r*.—These two stars are close together and their magnitudes were deduced from the same plates, the telescope being set at their mean R.A. and Dec. Ten plates were obtained, the Pole Field being photographed on each.

For N.P.S. 2*r*, first and second diffracted images were measured respectively on ten and eight plates, giving mean magnitudes $7^m{\cdot}89$ and $7^m{\cdot}93$. The magnitude adopted is $7^m{\cdot}91$, with a range of $0^m{\cdot}30$ and a mean discordance of $0^m{\cdot}10$.

The principal image of N.P.S. 4*r* was measured on ten plates and the first diffracted image on eight plates. The resulting mean magnitudes are $9^m{\cdot}07$ and $9^m{\cdot}14$ respectively. The magnitude adopted is $9^m{\cdot}11$, the ten plates having a range of $0^m{\cdot}28$ and a mean discordance of $0^m{\cdot}06$.

N.P.S. 3*r* and 5*r*.—The magnitudes of these two stars, which are close together, were also obtained from the same plates. Six plates were obtained on which the pair were compared directly with the Pole. Each star was at a distance of about 20′ from the centre of the plate. The principal images only were measured.

The magnitude deduced for N.P.S. 3*r* is $8^m{\cdot}93$, with a range from the six plates of $0^m{\cdot}16$ and a mean discordance of $0^m{\cdot}03$; for N.P.S. 5*r*, the value found is $10^m{\cdot}10$, with a range of $0^m{\cdot}22$ and a mean discordance of $0^m{\cdot}08$.

These two stars were also measured on certain of the bright star plates. From nine of these plates, a mean magnitude for N.P.S. 3*r* is found of $9^m{\cdot}03$, with a range of $0^m{\cdot}28$ and a mean discordance of $0^m{\cdot}06$. N.P.S. 5*r* was measured on five plates, which give a mean magnitude of $10^m{\cdot}07$, with a range of $0^m{\cdot}17$ and a mean discordance of $0^m{\cdot}06$. On these plates, the stars were about 45′ from the centre of the plate.

N.P.S. 5*r* was also measured on nine plates taken primarily for N.P.S. 1*r*, the star being at a distance of 45′ from the centre of the plate. These measures give a mean magnitude of $10^m{\cdot}27$, with a range of $0^m{\cdot}25$ and a mean discordance of $0^m{\cdot}07$.

The magnitude adopted for N.P.S. 3*r* is $8^m{\cdot}97$, and for N.P.S. 5*r* is $10^m{\cdot}13$.

12. *Pole Field.*—The stars of the Polar Sequence after N.P.S. 9, 3*s*, and 5*r* are all obtained from the Pole Field itself. The magnitudes of N.P.S. 10 and 11, which are brighter than 10^m, were primarily determined from the long-exposure Pole plates, by measuring the diffracted images. Provisional magnitudes were thus deduced, which were assumed in subsequent reductions. From all the plates on which the Pole Field was photographed, the magnitudes of the sequence stars in the field were read off. The mean values of the magnitudes so determined, with the number of plates utilised for each star, are given in the summary below. They carry the determination of the magnitudes of the Polar Sequence down to a limit of $13^m{\cdot}4$; on a few of the longer exposure plates some fainter stars were shown, but they were too close to the threshold of the plate to be measurable with accuracy. The present investigation, therefore, covers a range of about eleven magnitudes.

13. *Light Variation of Polaris.*—After the magnitudes of Polaris had been obtained from the bright star plates, an examination of the results was made as to whether they revealed any evidence of variation. The elements assumed were

$$M = \text{J.D. } 2418985{\cdot}936 + 3^d{\cdot}9681n$$
$$M - m = 1^d{\cdot}984.$$

No attempt had been made when taking the plates to secure a distribution well-balanced about the mean brightness, and it was found that unfortunately a large majority of the plates had been taken when the brightness was above the mean. Of the fourteen plates taken for N.P.S. 1 on which Polaris was measured, eleven were obtained when the star was brighter than the mean. Assuming a range of variation of $0^m{\cdot}10$, the results of these plates may be summarised as follows:—

	First Diff. Image.	Second Diff. Image.
Mean mag. from eleven plates	2·26 (6 plates)	2·20 (11 plates)
Correction to reduce to mean mag.	+ ·04	+ ·04
Mean mag. from three plates	2·35 (3 plates)	2·28 (3 plates)
Correction to reduce to mean mag.	− ·04	− ·04

The three plates obtained when Polaris was fainter than mean give a magnitude difference compared with those obtained when brighter than the mean of $+0^m{\cdot}09$ and $+0^m{\cdot}08$, using first and second diffracted images respectively. These differences, after applying the correction for variation, are reduced to $+0^m{\cdot}01$ and $0^m{\cdot}00$.

Polaris was also compared directly with N.P.S. 2 on nine plates. Of these, six plates were obtained when it was brighter than the mean.

	Magnitude of First Diffracted Image of Polaris *minus* Magnitude of N.P.S. 2.
	m
Mean of six plates	+ ·08
Correction to reduce to mean mag.	+ ·04
Mean of three plates	+ ·14
Correction to reduce to mean mag.	− ·04

The difference of $+^{m}\cdot06$, as between the plates taken when Polaris was faint and bright respectively, is therefore reduced, after correction for variation, to $-^{m}\cdot02$.

These results indicate that a range of variation in photographic magnitude is about $0^{m}\cdot10$.

14. *Summary of Results.*—The magnitudes obtained in the present investigation are summarised in the table below. The first column of this table gives the Harvard North Polar Sequence number; the second column gives the B.D. numbers of the brighter stars; the third and fourth columns give respectively the right ascension and declination for the epoch 1900; the fifth column contains the magnitudes adopted from the present work; and in the next two columns are given the differences between these magnitudes and those determined at Mount Wilson (F. H. Seares, *Mount Wilson Contributions*, No. 97) and the final revised values obtained at Harvard (*Harvard Annals*, 71, part 3) respectively. The eighth column gives the colour index determined at Mount Wilson. The ninth column gives the number of plates, from which the magnitudes of each star were determined in the present investigation. The numbers of plates for the bright stars are fewer than for the brighter stars in the Pole Field, but for these stars the weight of a single plate is greater.

Catalogue of Magnitudes.

N.P.S. No.	B.D. No.	R.A. 1900.	N. Dec. 1900.	Greenwich Mag.	Greenwich *minus* M.W. (Unit $0^{m}\cdot01$.)	Greenwich *minus* H.C.	Colour Index (M.W.).	No. of Plates.
		h m	° ′	m			m	
1	86°·269	18 4·6	86 36·8	4·38	− 1	−10	0·02	14
2	85·383	22 21·3	85 36·3	5·24	− 6	− 4	0·02	11
3	86·344	23 27·8	86 45·3	5·77	− 6	− 4	0·27	15
4	86·272	18 7·8	86 59·6	5·87	− 4	−12	0·07	25
5	88·4	0 55·6	88 29·3	6·46	+ 1	− 3	−0·02	26
6	89·13	7 58·0	88 56·0	7·10	− 1	− 1	0·06	14
7	88·64	11 4·2	88 11·0	7·35	− 6	+ 4	−0·11	31
8	88·9	2 14·2	88 42·1	8·35	+ 3	+12	0·19	39
9	88·13	2 42·2	88 34·1	8·98	+10	+15	0·07	33
10	89·3	3 18·9	89 41·2	9·12	0	+10	0·05	108
11	89·18	11 35·0	89 29·1	9·82	+ 9	+27	0·20	114
12	89·25	13 51·5	89 28·9	10·09	0	+23	0·29	113
13	89·29	15 13·6	89 38·4	10·51	− 2	+21	0·16	116
14	89·1	0 34·0	89 51·2	10·88	−10	+23	0·44	109
15		7 43·2	89 45·3	11·27	+ 5	+19	0·33	96
16		20 55·2	89 43·2	11·56	− 6	+16	0·39	81
17		18 1·2	89 40·2	11·89	+ 2	+26	0·59	56
18		18 43·6	89 43·6	12·27	0	+21	0·38	37
19		17 33·6	89 52·2	12·66	− 3	+24	0·41	15
20		19 46·8	89 55·0	13·02	0	+29	0·51	11
21		17 21·6	89 44·6	13·39	+ 6	+43	0·85	9

Catalogue of Magnitudes—continued.

N.P.S. No.	B.D. No.	R.A. 1900.	N. Dec. 1900.	Greenwich Mag.	Greenwich *minus* M.W. (Unit $0^m\cdot01$.)	Greenwich *minus* H.C.	Colour Index (M.W.).	No. of Plates.
	°	h m	° ′	m			m	
1*s*	88·8	1 22·6	88 46·4	2·54	0	−15	0·46	23
2*s*	88·71	12 14·4	88 15·3	6·52	+ 7	+ 4	0·12	30
3*s*	87·107	12 13·9	86 59·5	6·65	+ 1	0	0·29	9
4*s*	89·12	8 30·5	89 32·2	10·37	+12	+25	0·42	127
5*s*	89·37	15 29·3	89 52·8	11·03	− 6	+22	1·03	108
6*s*	89·26	12 53·2	89 41·7	11·36	− 2	+25	0·65	95
7*s*		15 24·0	89 53·0	12·55	− 6	+10	0·51	22
1*r*	87·51	6 53·7	87 12·3	6·72	+11	− 8	1·52	11
2*r*	88·112	19 22·5	88 59·3	7·91	− 3	+ 4	1·59	10
3*r*	88·76	13 4·5	88 11·2	8·97	+ 1	+22	1·40	15
4*r*	88·114	19 43·3	88 41·1	9·11	−11	+ 1	0·96	10
5*r*	89·22	12 51·5	88 54·1	10·13	0	+22	1·48	20
6*r*	89·9	6 38·9	89 28·1	10·58	+12	+28	1·25	116
7*r*	89·35	15 55·1	89 46·2	10·96	+ 2	+26	1·04	111
8*r*	89·31	16 49·3	89 38·6	11·43	− 1	+23	1·00	93
9*r*		14 51·2	89 34·2	11·95	...	+25	...	48
10*r*		13 8·0	89 42·7	12·60	...	+25	...	20
11*r*		14 58·4	89 53·3	13·29	+ 7	+32	1·16	8

15. The zero of the magnitudes given in the fifth column of the preceding table was chosen to give agreement with the Mount Wilson scale for the principal and supplementary sequence stars between $10^m\cdot0$ and $13^m\cdot5$. The zero of the Mount Wilson series was chosen so as to agree with the Harvard scale at the sixth magnitude.

The differences in the sixth column in no case exceed $0^m\cdot12$, and indicate a very close parallelism between the results obtained at Mount Wilson and in the present investigation. The differences from Harvard magnitudes given in the seventh column show a progressive variation. The two series agree at about the sixth magnitude, but for brighter stars the Harvard magnitude is too faint, whilst for stars fainter than sixth magnitude the Harvard magnitude is too bright.

The present investigation, therefore, confirms the Mount Wilson photographic scale between the limits of magnitude $2^m\cdot5$ and $13^m\cdot4$.

The magnitudes of the sequence stars between 10^m and 16^m have previously been determined at Greenwich with the 26-in. refractor; Seares found that the differences between these magnitudes and those found for the same stars with the 60-in. reflector at Mount Wilson showed a close correlation with the colour of the star. If a red star with colour index of $1^m\cdot0$ is measured with the 26-in. refractor $0^m\cdot27$ brighter than a blue star with colour index $0^m\cdot0$, the two stars are found with the reflector to be of equal magnitude.

No marked colour difference between the results obtained with the

13-in. refractor and the 100-in. reflector is shown by the residuals in the sixth column of the above table. A least-squares solution was made, giving equal weight to each star, and representing the residuals in the form $a + b(m - 10) + cC$, C being the colour index and m the magnitude of the star. The second term would allow for a difference in light-ratio between the two series of results such as might have occurred, for instance, had an incorrect value of the grating interval been adopted in the present investigation. The following relation was thus found:—

$$\text{M.W.} - \text{Gwh.} = +0^{m}\cdot01 + 0^{m}\cdot001(m - 10) - 0\cdot04\text{C}.$$

The smallness of the coefficient of the second term indicates that the two scales are in almost perfect agreement as regards light-ratio. There is a small colour coefficient, in a sense opposite to that obtained with the 26-in. refractor but similar to—though smaller than—the value found by Seares for the Potsdam 80-cm. refractor ($-0\cdot06$).

The colour equation of the 13-in. refractor is therefore not greatly different from that of the reflector, a result of value in photometric work. For a difference in colour index of one magnitude there is a relative difference between the two instruments of only $0^{m}\cdot04$.

Applying this correction to the residuals (Greenwich minus Mount Wilson), to eliminate the small relative colour equation, the sum of the squares of the residuals was reduced by about 10 per cent.

16. *Summary.*—The photographic magnitudes of all the stars in the North Polar Sequence, down to a limit of $13^{m}\cdot4$, have been determined. The 13-in. astrographic refractor was used in conjunction with a parallel wire grating: this grating gives magnitude differences between central and first and second diffracted images of $2^{m}\cdot83$ and $3^{m}\cdot48$ respectively. The results obtained are summarised in the table accompanying this paper: they confirm the magnitude scale determined at Mount Wilson with the 60-in. reflector. Comparison of the two series of results indicates very close agreement between the two series in light-ratio. It is also found that the colour equation of the 13-in. refractor does not differ greatly from that of the reflector.

Stellar Parallaxes determined at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The following list is a continuation of the results obtained with the 26-inch telescope.

The method of measurement, by means of an auxiliary scale plate, as described in the *Monthly Notices* for Nov. 1920, has been continued and appears to be quite satisfactory.

In the column Proper Motion the first is derived from meridian observations; the second, in R.A. only, is the relative proper motion derived from the parallax plates. (Both are expressed in arc of a great circle.)

In calculating the parallax, the relative proper motion is used, *not* the meridian proper motion.

The Probable Error has been calculated by the formula

$$r = 0{\cdot}6745 \times \sqrt{\frac{[vv]}{m-\mu}} \times \frac{1}{\sqrt{w}}.$$

The first given is the P.E. of the parallax, the second is the P.E. of the relative proper motion.

Generally six comparison stars of mean magnitude 11–12 have been used, the magnitude of the parallax star being reduced to equivalence by means of the rotating sector.

Seven of the stars, Nos. 11, 13, 15, 18, 27, 31, and 41 have already appeared in the published lists. The values given here are not new determinations, but revised results with additional material.

No.	Name and B. D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. Dec. 1900·0.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s	″	″	″	″	
1	Brad. 34	6·5	G5	0 24 30	+·335	−·016	+·020	±·011	15
	+76° 10	6·9		+76° 28′	+·336			±·003	
2	54 Cass.	6·7	F8	2 0 30	+·294	−·245	+·015	±·008	25
	70° 163	6·9		+71° 4′	+·306			±·003	
3	67° 209	9·0		2 23 14	+·167	−·017	+·005	±·008	21
		9·4		+67° 59′	+·129			±·002	
4	L.F. 526	6·9	F5	3 12 58	+·188	−·145	+·005	±·005	12
	73° 180	7·1		+73° 49′	+·195			±·002	
5	Br. 671	7·0	F8	4 52 42	+·074	−·347	+·032	±·007	16
	66° 370	6·7		+66° 40′	+·060			±·003	
6	O.A. 5726	8·0	F8	5 14 0	+·221	−·151	+·018	±·008	16
	63° 577	7·5		+64° 2′	+·172			±·002	
7	L.F. 1217	7·7	F5	7 55 47	−·183	−·243	+·046	±·010	17
	68° 518	8·2		+68° 40′	−·186			±·004	
8	L.F. 1221	8·0	Go	7 57 13	−·199	−·445	+·012	±·009	15
	72° 395	8·4		+72° 13′	−·229			±·002	
9	L.F. 1391	7·5	Go	8 51 43	−·050	−·178	+·022	±·009	17
	76° 335	8·2		+76° 48′	−·057			±·003	
10	O.A. 9788	8·3		9 16 10	−·945	−·079	+·047	±·011	13
	76° 351	9·7		+76° 22′	−·335			±·004	
11	O.A. 13635	9·1	G5	13 21 46	−·267	+·110	−·015	±·010	20
	68° 724	10·2		+68° 42′	−·208			±·002	
12	O.A. 15127	9·0		15 3 10	−·229	−·021	+·036	±·008	16
	65° 1033	9·8		+65° 12′	−·266			±·003	

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. Dec. 1900·0.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s	″	″	″	″	
13	O.A. 15246	8·9		15 9 37	−·106	+·183	+·022	±·011	20
	76° 552	9·4		+76° 34′	−·117			±·003	
14	O.A. 15254	8·6	Ko	15 11 31	−·216	+·080	+·011	±·010	19
	72° 674	9·2		+72° 13′	−·188			±·003	
15	L.F. 2684	8·2	Go	15 31 48	+·225	+·039	+·003	±·010	15
	82° 456	7·9		+82° 14′	+·221			±·006	
16	Gr. 2275	6·5	G5	15 34 59	−·197	+·126	+·048	±·012	17
	80° 480	7·0		+80° 47′	−·206			±·004	
17	Gr. 2276	8·3	G5	15 35 12	−·180	+·121	+·037	±·013	15
	80° 481	7·9		+80° 47′	−·206			±·003	
18	O.A. 15755	9·0		15 49 47	+·095	−·317	+·031	±·007	19
	74° 634	10·1		+74° 43′	+·102			±·002	
19	O.A. 15927	9·1		16 3 3	−·225	+·151	+·027	±·006	22
	68° 861	10·0		+68° 48′	−·224			±·002	
20	O.A. 15951	9·3		16 4 51	−·118	+·178	+·003	±·013	15
	67° 925	9·7		+67° 3′	−·124			±·004	
21	L.F. 2762	7·3	G5	16 15 32	−·008	−·315	+·038	±·008	21
	71° 775	8·4		+71° 11′	·000			±·003	
22	L. 29917	8·6		16 16 32	−·488	+·092	+·088	±·007	19
	67° 935	9·5		+67° 29′	−·490			±·002	
23	η Ursæ Min.	5·3	Fo	16 20 25	−·066	+·252	+·041	±·009	18
	76° 596	5·5		+75° 59′	−·047			±·004	
24	L. 30699	7·6	G5	16 42 51	−·272	+·434	−·042	±·008	24
	68° 883	8·0		+68° 16′	−·278			±·002	
25	Gr. 2391	6·0	F2	16 47 33	+·056	+·206	·000	±·008	24
	77° 634	6·4		+77° 41′	+·063			±·003	
26	68° 901	9·0		16 56 38	−·064	+·267	+·013	±·009	16
		8·9		+68° 10′	−·046			±·005	
27	L. 32047	6·5	Ko	17 25 18	−·539	+·008	+·065	±·008	30
	67° 1014	6·8		+67° 23′	−·523			±·002	
28	L. 32151	7·5		17 29 7	−·005	−·207	+·015	±·011	24
	63° 1358	7·8		+63° 56′	−·005			±·002	
29	78° 600	8·9		17 34 57			+·004	±·008	26
		9·7		+78° 27′	+·007			±·002	

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. Dec. 1900·0.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s	"	"	"	"	
30	78° 601	9·0		17 35 25			−·002	±·008	27
		10·3		+78° 24′	+·006			±·002	
31	O.A. 17415	9·1		17 37 0	−·373	−1·276	+·244	±·010	28
	68° 946	10·3		+68° 25′	−·354			±·003	
32	O.A. 17433	6·5	F8	17 37 31	−·043	−·220	+·013	±·009	16
	69° 933	6·7		+69° 37′	−·048			±·002	
33	ω Draco.	4·8	F5	17 37 32	+·010	+·327	+·005	±·011	17
	68° 949	5·2		+68° 48′	+·036			±·002	
34	71° 851	9·0		17 40 4	−·147	+·291	+·059	±·013	26
		9·9		+71° 22′	−·105			±·002	
35	ψ^1 Draco.	4·8	F5	17 43 43	+·015	−·267	+·057	±·012	17
	72° 804	5·2		+72° 12′	+·025			±·010	
36	ψ^2 Draco.	6·2	F5	17 43 45	+·021	−·278	+·030	±·014	17
	72° 805	6·4		+72° 12′	+·038			±·011	
37	78° 612	8·0	G	17 53 6	−·103	+·173	+·006	±·010	19
		9·0		+78° 25′	−·122			±·008	
38	36 Draco.	5·0	F2	18 13 19	+·345	+·029	+·057	±·013	14
	64° 1252	5·5		+64° 22′	+·342			±·007	
39	65° 1254	9·4		18 15 40			−·011	±·008	21
		9·7		+65° 42′	+·003			±·006	
40	68° 986	9·5		18 16 35	+·131	+·426	+·001	±·009	22
		10·6		+68° 36′	+·160			±·007	
41	χ Draco.	3·8	F5	18 22 52	+·521	−·369	+·134	±·009	16
	72° 839	4·3		+72° 41′	+·464			±·017	
42	84° 417	9·5		18 36 49	+·156	+·164	+·026	±·012	19
		10·5		+84° 8′	+·146			±·003	
43	L. 34968	8·0	G5	18 38 30	−·131	+·193	−·001	±·010	23
	67° 1087	8·0		+67° 2′	−·124			±·002	
44	P. XVIII. 273	7·5	Go	18 49 48	+·325	+·047	+·005	±·014	25
	74° 792	7·6		+74° 36′	+·323			±·003	
45	71° 943	9·0		19 16 3	+·089	+·184	+·029	±·008	12
		9·9		+71° 21′	+·108			±·005	
46	65° 1406	8·5	F8	19 48 28	+·055	+·223	+·034	±·007	18
		8·6		+65° 42′	+·057			±·003	

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. Dec. 1900·0.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s	"	"	"	"	
47	76° 785	9·0		20 13 51	+·080	+·493	+·056	±·008	21
		9·5		+76° 55′	+·072			±·004	
48	Gr. 3150	6·2	F8	20 16 32	+·474	+·293	+·071	±·007	21
	66° 1281	6·3		+66° 32′	+·452			±·005	
49	74 Draco.	6·1	K0	20 35 15	+·072	+·216	+·043	±·009	14
	80° 660	6·7		+80° 45′	+·005			±·022	
50	L.F. 3638	7·8	G5	20 52 22	+·417	+·566	+·045	±·010	19
	74° 889	8·3		+74° 23′	+·418			±·003	
51	80° 694	9·5		21 24 0	−·056	−·036	+·055	±·011	18
		10·1		+81° 9′	−·078			±·002	
52	L.F. 3880	8·0	G5	21 30 51	+·221	+·095	−·004	±·011	18
	81° 742	8·5		+81° 18′	+·221			±·002	
53	64° 1582	9·0		21 39 38	+·141	+·154	−·021	±·010	18
		9·7		+64° 28′	+·092			±·003	
54	C. 3505	8·0	K2	22 48 32	−·161	+·125	+·031	±·008	22
	82° 704	8·1		+83° 10′	−·176			±·002	

Note on the Question of Secondary Oscillations in the Light Curves of Cepheid Variables. By Frank C. Jordan.

(*Communicated by the Secretaries.*)

There have been published in recent years in the *Monthly Notices* by Martin and Plummer a considerable number of light curves of short-period variable stars of the δ Cephei type. In every case these curves have been drawn with sinuosities on the downward slope, the result of an elaborate harmonic analysis of the observations. A number of the same stars have been studied at the Allegheny Observatory, and a comparison of the methods of observation and the results may not be out of place. The observations of Martin and Plummer have been made with the 15-inch reflector of the Dunsink Observatory. The images are over-exposed and the measures of diameter made, not on the edges of the images, as is the usual method, but on the indefinite dividing line between a dense nucleus and the less dense surrounding part of the image. The claim is made that the results are very accurate (*M.N.*, **73**, 166).

I have had considerable experience in measuring star-images as obtained with the 2-foot reflector of the Yerkes Observatory. These images, especially the ones obtained on isochromatic plates with a colour filter, have very definite edges on which the micrometer wire can be set with a good degree of accuracy. Yet with such good images errors of 0·1 mag. are not uncommon. With the images obtained at Dunsink I

should think that errors of 0·2 mag. would be not at all rare. Their exposures cover a very appreciable part of the star's period, and would have a tendency to smooth out any sinuosities even if these should exist.

I do not believe the evidence of the Dunsink observations is strong enough to prove the existence of these irregularities, nor do I believe that harmonic analysis of such observations leads to the true shape of the curves. As evidence of this I offer for comparison some curves obtained at Allegheny. The extra-focal method is used here, and the resulting expanded images measured in a Hartmann microphotometer, the probable error of a measure being less than 0·01 mag. This by no

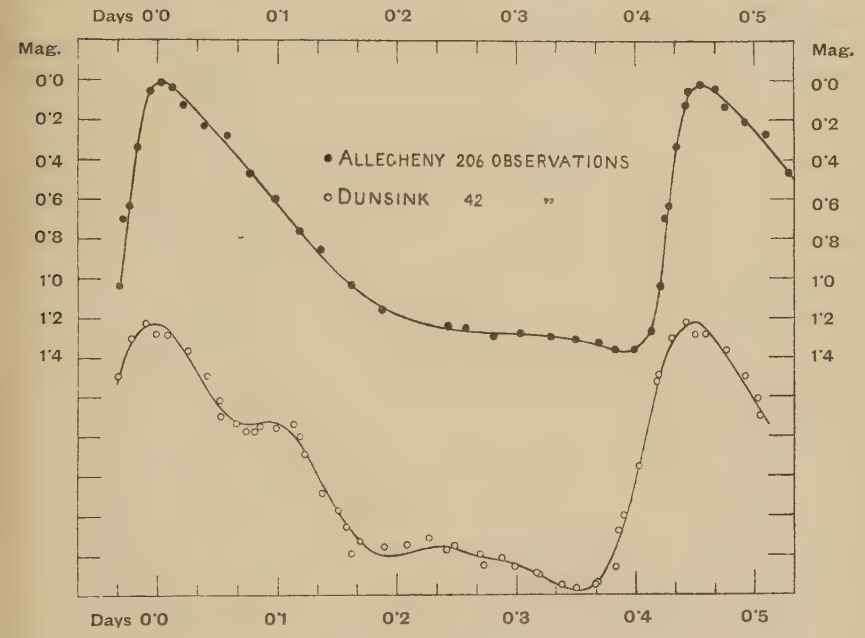


Fig. 1.—RR Leonis.

means implies that the real error of the magnitudes obtained is as small as this, for there are other sources of error: lack of uniformity of film, which probably affects extra-focal more seriously than focal images; a possible anomaly in a single image; the effect of colour, and still other factors. However, I believe this method of obtaining magnitudes is more accurate than any except that of the photo-electric cell. It is certainly more accurate than the measurement of star-diameters, however these may be made.

The first figure shows the curves of RR Leonis, whose period is 0·452372 days (*M.N.*, 81, 458). The Dunsink exposures are 40 to 60 minutes, one-fifteenth to one-tenth of the period. The Allegheny exposures are 4 minutes. In this figure the dots represent normal places derived from the 206 Allegheny observations. The 42 circles on

the Dunsink curve are single observations. The difference between the curves is obvious, and the four harmonic terms in the equation of the Dunsink curve have no meaning when applied to the Allegheny curve. The hump at phase 0·1 day is not even suggested in the latter curve.

A more striking example is seen in fig. 2, the curves of SU Draconis, whose period is 0·6604 days. There seems to be a hump in the Allegheny curve. However, I am inclined to think it can rather be attributed to a photographic effect than to any peculiarity of the curve itself. It happens that this point is the one most sparsely observed of any on the curve. It is the junction point of two sets of observations, the parts

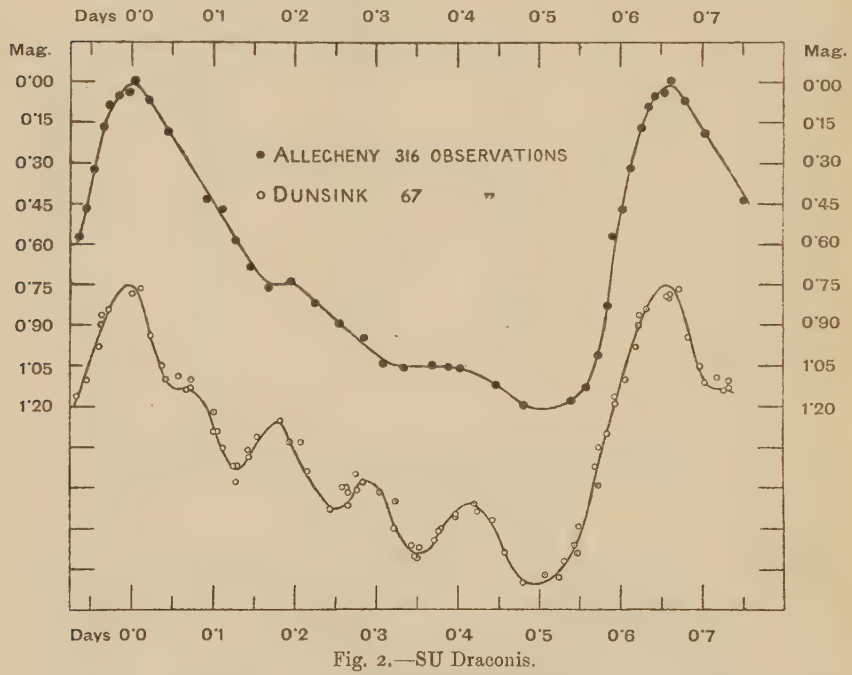


Fig. 2.—SU Draconis.

of the curve just before and just after this phase being covered by single night's observations separated nearly a month in time. If the curve were drawn without this hump, the residuals would still be reasonably small. The curves are derived from 316 and 67 observations respectively. The exposure times are 2 and 35 minutes respectively.

The question may arise as to whether the formation of normal points may not tend to smooth out any possible irregularities in the curves. As an answer to this, fig. 3 shows the down slopes of the curves of SW Draconis, whose period is 0·569662 days. The Allegheny observations are not sufficient to give the whole curve. I have therefore selected three sets of plates covering the down slope, each set a single night's observation. The different symbols indicate the different sets of plates. The greatest residual from a smooth curve is about 0·05 mag. There is

no indication whatever of any secondary oscillations. At phase 0·4 days in the Dunsink curve individual observations differ by more than 0·3 mag., and no harmonic analysis however complicated can fit a curve to these observations.

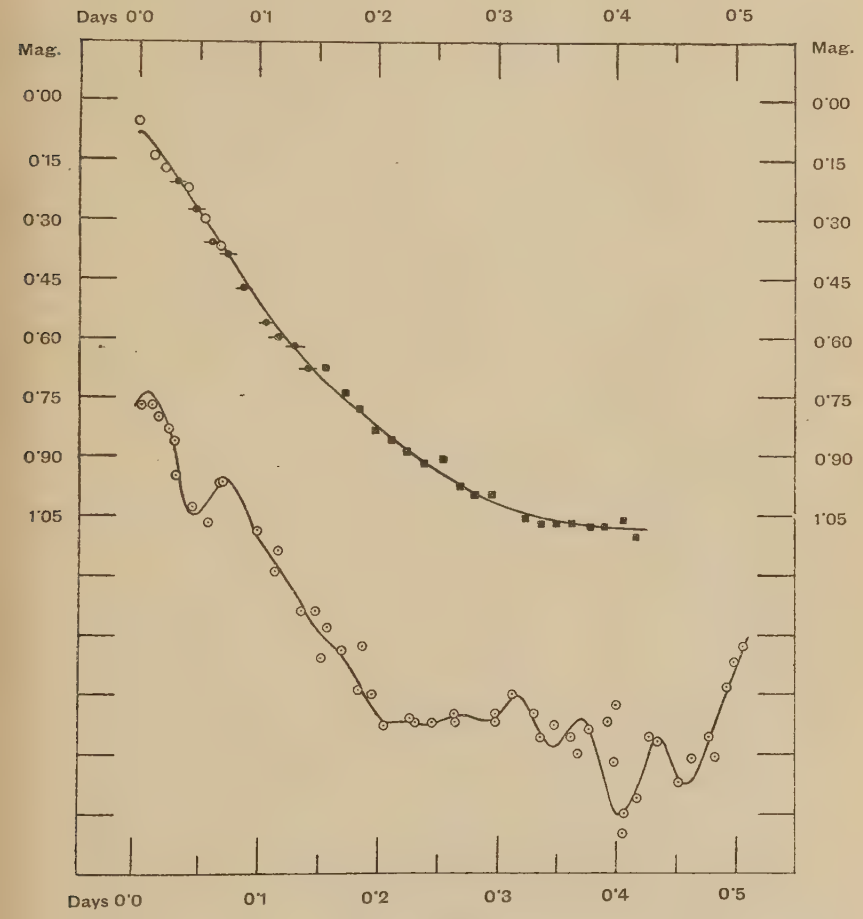


Fig. 3.—SW Draconis.

The longest Dunsink exposures are 55 minutes, or one-fifteenth of the period. The sinuosities as computed from the harmonic equation are some of them as small as this exposure time.

Other curves might be given, but these are sufficient to show that the evidence is strongly in favour of smooth curves for Cepheid variables.

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On Mr. Jordan's Note. By H. C. Plummer, F.R.S.

1. This note, though very far from giving a complete or final answer to the particular but undefined question to which it refers, is nevertheless to be welcomed in certain aspects. It is based on a method of observation which to all appearance is exceedingly promising. I am not aware of a single instance in which the light curves of this class of variable stars, as determined by independent observers, show a satisfactory accordance. Too commonly a most elastic limit has been allowed to the accuracy of the observations, and observers have been content to claim a general resemblance even when this quality is so vague as to have little value. In so far as Mr. Jordan discards this facile optimism he carries me with him. He will not say there is agreement when the discrepancies are obvious. In claiming boldly a certain order of accuracy for his own observations, he is doing no more than an observer ought to do, though the burden of proof rests with him.

2. If the discrepancies between the Allegheny and the Dunsink curves are to be traced to their true origin, we must at least understand that a highly complex problem confronts us. It is not quite such a simple matter as Mr. Jordan conceives it. There are at least four important questions to be discussed:—

(1) The accuracy of the Dunsink observations.
(2) The accuracy of the Allegheny observations.
(3) The quantity actually measured in the two cases, which is a function of the equipment and method employed.
(4) The behaviour of the star from period to period.

It is obvious that (1) and (2) can be confronted and brought into direct conflict only when (3) and (4) have been decided in a particular way. So little has Mr. Jordan appreciated the bearing of these points, that he has omitted to inform us what kind of instrument (refractor or reflector) he has used, as well as many other pertinent details, and at what date his observations were made. He has thus ignored all the physical factors in his problem, and then found an easy solution of all difficulties by assuming a high order of accuracy for his own measures and a low order for Mr. Martin's measures. It is a pity that scientific truth cannot always be arrived at in so simple a fashion.

3. Without unwarrantable assumptions the relative accuracy of the two series cannot be settled in this off-hand way. It is certainly desirable as a preliminary that this should be ascertained by independent investigations. For the accuracy of the Allegheny results no evidence is furnished beyond a bare assertion. I have no wish to disparage these results, which in themselves seem excellent. But a critic places himself in a poor position when he does not furnish the evidence for his own claim. On the other hand, much material bearing on the accuracy of the Dunsink measures has been published. All this is set aside with the remark that measures of diameter with images of short exposure give inaccurate results. The remark is wholly irrelevant. As a matter of fact, it is precisely the conclusion to which Mr. Martin was led by experience ten years ago. It ought to be sufficiently evident that nobody

would spend fifty minutes on a single exposure when an equally good result could be obtained in ten minutes or less.

4. Mr. Jordan seems to be under some misapprehension. The method of harmonic analysis has nothing whatever to do with the estimated accuracy of the Dunsink observations. The relation, in fact, is precisely the opposite: the estimated accuracy sets the limit to which it is worth while to carry the harmonic analysis. Also the analysis has nothing to do with proving the existence of the so-called secondary oscillations. It provides merely an alternative representation of the graphical form. At the most it substitutes a more definite method of drawing the interpolated curve, when scattered observations leave a wider element of choice in freehand drawing.

5. The quantity to be measured may be represented by the integral

$$\int \mathrm{ML}d\lambda$$

or a simple function of it. Here $Ld\lambda$ is the amount of light received from the star between wave-lengths λ and $\lambda + d\lambda$, and the factor M is a function of λ which represents both the proportion of this light transmitted to the recording apparatus (here the photographic plate) and the ratio of the response of the apparatus to the stimulus, or, in one word, the sensitiveness. Comparable results from independent series of observations can be expected only in very special circumstances. If the function $\mathrm{L}(\lambda, t)$ is of the form $\mathrm{L}_1(t)\mathrm{L}_2(\lambda)$, then $\mathrm{L}_1(t)$ is measured by two observers even if they use apparatus with different functions of sensitiveness $\mathrm{M}_1(\lambda)$ and $\mathrm{M}_2(\lambda)$. But this means that $\mathrm{L}_2(\lambda)$ is a curve independent of the time. In other words, the quality of the light, though not its intensity, remains unchanged. Now this is certainly not the case in the present problem. On the other hand, if $\mathrm{M}(\lambda)$ be the same function for two observers, their results should be comparable whatever the function $\mathrm{L}(\lambda, t)$ may be. But Mr. Jordan has made no attempt to deal with this primary consideration. Now it is fairly certain that visual and ordinary photographic observations differ widely. In the absence of evidence to the contrary, agreement between the Allegheny and the Dunsink results is no more to be expected than in other cases where different methods of measurement have been employed.

6. This consideration would hold even if the observations on the two sides had been made simultaneously. But though, in defiance of accepted convention, one set is entirely undated, it is sufficiently certain that the observations are very far from being simultaneous. For the two stars SU and SW Draconis several thousand periods have probably elapsed between the two sets of observations compared. Hence a comparison can be made only on the assumption that strict periodicity has been maintained throughout, so that

$$\mathrm{L}(\lambda, t) = \mathrm{L}(\lambda, t + \mathrm{EP}),$$

where P is the period of the star and E is any integer. But this assumption is almost certainly untrue. There is much material suggesting that the period is not always constant, and further, that in different periods the variations are far from being reproduced with faithful similarity.

Here Mr. Jordan may fairly reply that in making the Dunsink curves follow the individual observations closely this very assumption, which is now criticised, has equally been made. That is perfectly true. But it was a matter of necessity in the circumstances, not a matter of choice. The Dunsink method, especially in a bad climate, is painfully slow and laborious. In order to explore a certain field of investigation and to deal with a fair number of stars, is was necessary to be content in each case with the bare minimum of observations required to outline the light curve. The method is unsuitable for the discussion of constancy of period and casual departures from a mean light curve.

7. On the whole, the majority of the stars discussed at Dunsink give little reason to suspect that the deviations from a mean curve are very serious. But apart from similar work on these questions done elsewhere, unstable variation was suspected in at least two instances. One of these is precisely SW Draconis, to which Mr. Jordan refers, and the other XZ Cygni. It was natural in the first place to include fresh objects in the working programme, and only to repeat work already done when a certain amount of ground had been covered. But a repetition of the work on these stars, as opportunity offered, had been contemplated for some time, and perhaps Mr. Martin will consider the question of making further series of observations. The star SU Draconis was observed at Dunsink nine years ago, and the prima-facie inference from Mr. Jordan's comparison with his own presumably later observations is that the character of the variations has changed in the interval. There would be nothing inherently surprising in such a result, and Mr. Martin may think it worth while to make the test by repeating his observations. In the case of RR Leonis the observations contrasted belong presumably to much more nearly the same epoch. It is impossible to make a close comparison, but to all appearance the two curves can be brought into excellent agreement over three-quarters of the period. This makes the outstanding discrepancy the more remarkable, and by itself throws strong doubt on Mr. Jordan's easy explanation.

41 *Half Moon Street, W.*1 :
1921 *October* 22.

The Spectrum of Nova Cygni III., 1920. By F. J. M. Stratton, D.S.O., M.A.

(*Communicated by the Director, Solar Physics Observatory, Cambridge.*)

This paper contains a discussion of the photographs of the spectrum of Nova Cygni III., 1920, taken at Cambridge on 1920 August 26, 29 and September 7, 10, 11, 12, 16, 19, 26. The first two were taken by Professor Newall with the slit-spectrograph attached to the Huggins refractor, the remainder were taken by Mr. Stratton and Mr. Manning with a two-prism slit-spectrograph attached to the Newall telescope. The series of plates is too incomplete to allow of a full study of the history of the changes in the nova. Details are here published which

may be useful for comparison with other novæ or with material on this nova obtained elsewhere. Further detailed measures—or the plates themselves—can be readily made available for any observatory which may be making a complete study of this star.

I. *The Absorption Spectrum.*

The displaced absorption spectrum provides a riddle which can only be safely read when the plates investigated provide a close sequence. The hydrogen absorptions are multiple and very complex, but the great majority of the stronger absorption lines measured on the plates can be simply interpreted as metallic absorption lines displaced from their normal positions by amounts proportional to their wave-lengths. If we use the displacement factor $10^4\delta\lambda/\lambda$ as the simplest measure of the displacement (the equivalent velocity of approach in km. per sec. is given on multiplying by 30), we get the following figures for the usual nova group of enhanced spark lines of titanium and iron :—

TABLE I.

Date.	No. of Lines.	Displacement Factor.
Sept. 10	15	25
11	14	25
12	15	24
16	8	27
19	5	27

There can be little doubt of the substantial accuracy of the identification of these lines on September 10, 11, 12 ; and it is extremely likely that the identifications on September 16, 19 are also correct. A sample is given for September 11.

TABLE II.

Measured W.L.	Presumed Source.	Corresponding Displacement Factor.
4168·2	4179·0 pFe	26
4222·8	4233·3 pFe	25
4292·7	4303·3 pFe	25
4388·3	4399·9 pTi	24
4405·8	4417·0 pFe	25
4432·8	4444·0 pTi	25
4457·9	4468·7 pTi	24
4497·5	4508·5 pFe	24
4504·6	4515·5 pFe	24
4511·1	4522·8 pFe	25
4523·1	4534·1 pTi	25
4553·3	4563·9 pTi	23
4560·6	4572·2 pTi	25
4572·8	4584·0 pFe	24

The identifications are supported by the presence of lines of other elements found in the same stellar spectra and found here with the

same displacement factor. Thus we have 4247·0 Sc with the displacement factor 25 on September 10; 4471·6 He with displacement factor 25 on September 11; and several enhanced lines of chromium with factors 24, 25, and 22 for September 10, 11, 12 respectively. One of the separate components of the broad complex hydrogen absorptions also in general agrees fairly well in displacement with the metallic lines. The results are given below:—

TABLE III.

Date.	H_δ.	H_γ.	H_β.	Metallic Lines.
Sept. 10	23	18 30	...	25
11	26	28	...	25
12	25	22 28	26	24
16	...	24 28	23 27	27
19	27	26	26	27

The hydrogen absorptions are more complex than the metallic ones. They suggest jets or shells of outrushing hydrogen moving with greater velocity than the heavier vapours, though some of the hydrogen mingles with the other gases. The details of the displacement factors for hydrogen found on the Cambridge plates are given below:—

TABLE IV.

	Aug. 26.	29.	Sept. 10.	11.	12.	16.	19.
				60	60		
				50			
				40	43		
$H\delta$						36	
	30		23	26	25	27	
					18		
						13	
			71?		66?		
			60	59	59	59	
			53	51	52	51	
			49	48			
$H\gamma$			45	40	42	41	43
						35	35
	32	29	30	28	28	28	26
					22	24 21	
			18			13	
						32	
$H\beta$	30				26	27	26
						23	

On a comparison of these results with the radial velocities published in other investigations, the following points of interest arise:—Harper's values * for the displacement factor, ranging from 22 on August 24 through 28 on August 25, 37 on August 29, to 40 on September 6, possibly belong to blends of the components measured separately on the Cambridge plates. Alternatively, they may mean that one component has moved out steadily. This component would fit in with the one with factor 31 measured at Cambridge on August 26. According to this view, the component with factor 27 present on September 11 is a fresh absorption, the previous absorption having disappeared except for the hydrogen lines. This would be parallel to the changes observed in Nova Geminorum on the Cambridge and Allegheny series of plates; and it is supported by the group of absorptions measured on September 6 at Victoria. The absorptions measured at λλ4424, 4450, 4472, 4497, 4555 would, with Harper's factor, correspond to the enhanced lines of iron and titanium at λλ4444, 4468, 4489, 4515, 4572. It is impossible to settle between these two possibilities without a study of plates of intermediate dates and of fairly high dispersion.

Dr. Lockyer's velocities † cannot be accepted now. They serve the purpose of showing the danger of using Orbinski's method when the absorption lines are so complex as they are found to be in novæ. The multiple nature of the absorption lines and the values of the displacement factors in Table IV. are confirmed by the Lick observations. Wright gives ‡ factors 60, 52, 44, 36, 30, 26 for the mean of the values for H_β, H_γ, and H_δ on September 17, 18.

Some of the other absorption lines have shown signs of a complex structure of the same nature as that found for the hydrogen lines. But though occasional lines, otherwise not identified, can be explained in this way, the evidence is incomplete and inconclusive. There are a certain number of lines about λ4600, but they are not steady enough in position nor numerous enough to connect safely with any definite origins. The Cambridge plates provide no evidence of the emergence of any other type of spectrum than that of α Cygni, which accounts when displaced with factors given in Table I. for every line measured on as many as three plates.

II. *The Bright-Band Spectrum.*

The difficulty of measuring the edges of bright bands in the spectra of novæ is by now a very familiar one. That together with the manner in which the bands are cut into by displaced absorptions from other sources makes the evidence for identification of the bands a little less certain than for the absorption lines. However, the general features of these spectra (viz. a series of broad bright bands of widths approximately proportional to their wavelengths, and with centres slightly displaced from the normal positions)

* *Publ. of the Dominion Astrophysical Observatory*, **1**, 271.
† *M.N.R.A.S.*, **81**, 46.
‡ *Pub. Ast. Soc. Pac.*, **32**, 276.

are common to the novæ and reappear in this case. The lines represented are as usual the Balmer series of hydrogen and enhanced spark lines of various metals. The following table gives the positions of the principal bands whose origins can be clearly traced to a single line or to a close double. Blends of separated lines, though undoubtedly present, are generally ignored.

TABLE V.

(Centres of Bands about 20A. wide.)

Origin \ Date.	Aug. 26.	29.	Sept. 7.	10.	11.	12.	16.	19.	26.
Hδ		4104·5		4101·1	4102·5	4102·7		4101·4	
pFe 4233·3		4235·7		4232·6	4231·9	4233·3		4232·1	
Hγ	4343·3	4343·3	4341·2	4341·5	4342·3	4341·2	4340·8	4342·0	4342·3
pTi {4395·2, 4399·9}	4397·4	4396·0		4395·5	4394·9				
pFe 4417·0, pTi 4417·9	4418·3	4417·2			4418·9	4416·8			
pTi 4444·0	4444·1	4446·6		4445·4	4445·9	4445·6			
pTi 4468·7		4467·0		4467·4		4467·5			
pFe 4489·3, pTi 4488·5					4490·5				
pFe 4549·6, pTi 4549·8					4549·7	4548·7			
pFe 4584·0	4583·3	4583·5		4584·1	4583·5	4583·2			
pTi 4805·2	4804·3								
Hβ		4859·9	4862·5	4861·5	4861·6	4862·6	4861·8	4861·5	4861·3
pFe 4924·1	4926·6			4924·4	4924·7	4925·0			
pFe 5018·6				5018·4	5017·8	5017·1			

The hydrogen bands mostly contain two maxima displaced to the red and violet of the undisplaced position of the origin. Tables of these maxima are given below. The widths of the maxima varied on different plates from about 5 to 8 Ångström units. As a general rule the violet maximum was slightly broader; the two maxima were of nearly equal intensity in the hydrogen bands, but a red maximum was occasionally measurable in the iron bands also.

TABLE VI.

Maxima in Bright Bands.

	Displacement to Red in Ångström Units.			Displacement to Violet in Ångström Units.		
	H_δ.	H_γ.	H_β.	H_δ.	H_γ.	H_β.
Aug. 26	10·2	9·8	6·7	7·7	3·9	4·2
29		9·5	8·0		5·9	5·6
Sept. 7		6·7	7·4		3·8	5·3
10		6·6	6·5		3·8	4·8
11	5·9	5·5	6·2			
12	5·3	5·6	5·9		3·6	5·5
19		5·4			3·3	
	pFe 4233·3.	pFe 4303·3.	pFe 4584·0.	pFe 4233·3.		
10	5·7			5·2		
11	4·6	4·8				
12		3·9	5·9			

There is a tendency for the maxima to be symmetrically placed with reference to the centre of the band, but the measures are too uncertain for any stress to be laid on this point.

It is hoped that the above data may be useful to other investigators who may be studying the changes in the spectrum of Nova Cygni III.

Recent Observations of Novæ. By W. H. Steavenson. (Plate 2.)

The following Novæ have been under regular observation during the summer and autumn of 1921: Novæ Ophiuchi (1848), Aquilæ (1918), Cygni (1920), Cygni (1876), Lacertæ (1910), and Persei (1901). Most of the observations were made with the 6-inch Wray refractor, presented by Mr. Drower, but, by the courtesy of the Astronomer Royal and others, occasional use was made of six other instruments. Owing to the peculiar nature of the light of the Novæ, estimates of their brightness are apt to vary systematically with the type of telescope employed, and, in the case of refractors, with differences in the colour correction of the objectives. I have therefore thought it best to record against each estimate a letter indicating the particular instrument used in making it, according to the following list:—

G = 28-inch Grubb refractor of the Royal Observatory.
B = 18-inch With reflector, belonging to the B.A.A.
P = 12½-inch Calver reflector, belonging to the Rev. T. E. R. Phillips.
A = 9-inch reflector, made by and belonging to Commander M. A. Ainslie.
R = 8-inch Cooke refractor, belonging to the Society.
M = 6-inch Cooke refractor, belonging to Mr. G. Merton.
S = 6-inch Wray refractor, belonging to the writer.

Nova Ophiuchi (1848).

Observed on 28 nights, between 1921 June 1 and Sept. 27. This star is of particular interest in view of the variations noted by Professor Barnard (*M.N.*, 81, 167), and recently dealt with by Professor Turner (*M.N.*, 81, 426). Following are the individual estimates :—

Date.	Mag.	Instrument.	Date.	Mag.	Instrument.
1921 June 1	12·1	S	1921 July 12	12·5	S
2	12·1	S	25	12·3	S
6	12·1	S	31	12·5	S
7	12·05	S	Aug. 2	12·45	S
8	12·05	S	3	12·4	M
9	12·05	S	6	12·5	S
11	12·3	G	8	12·6	S
28	12·8	M	9	12·65	S
30	13·0	S	27	12·65	S
July 4	12·75	S	29	12·65	S
5	12·6	S	Sept. 5	<12·3	S
8	12·8	S	7	12·7	S
9	12·6	S	8	12·9	B
10	12·5	S	27	<12·3	S

When plotted, these observations, taken up to Aug. 9, give clear evidence of variability in a period of about 50 days. The subsequent observations do not seem to confirm this period, but they are few in number and were made when the star was low and difficult to observe. They are certainly not entitled to the same weight as the earlier estimates.

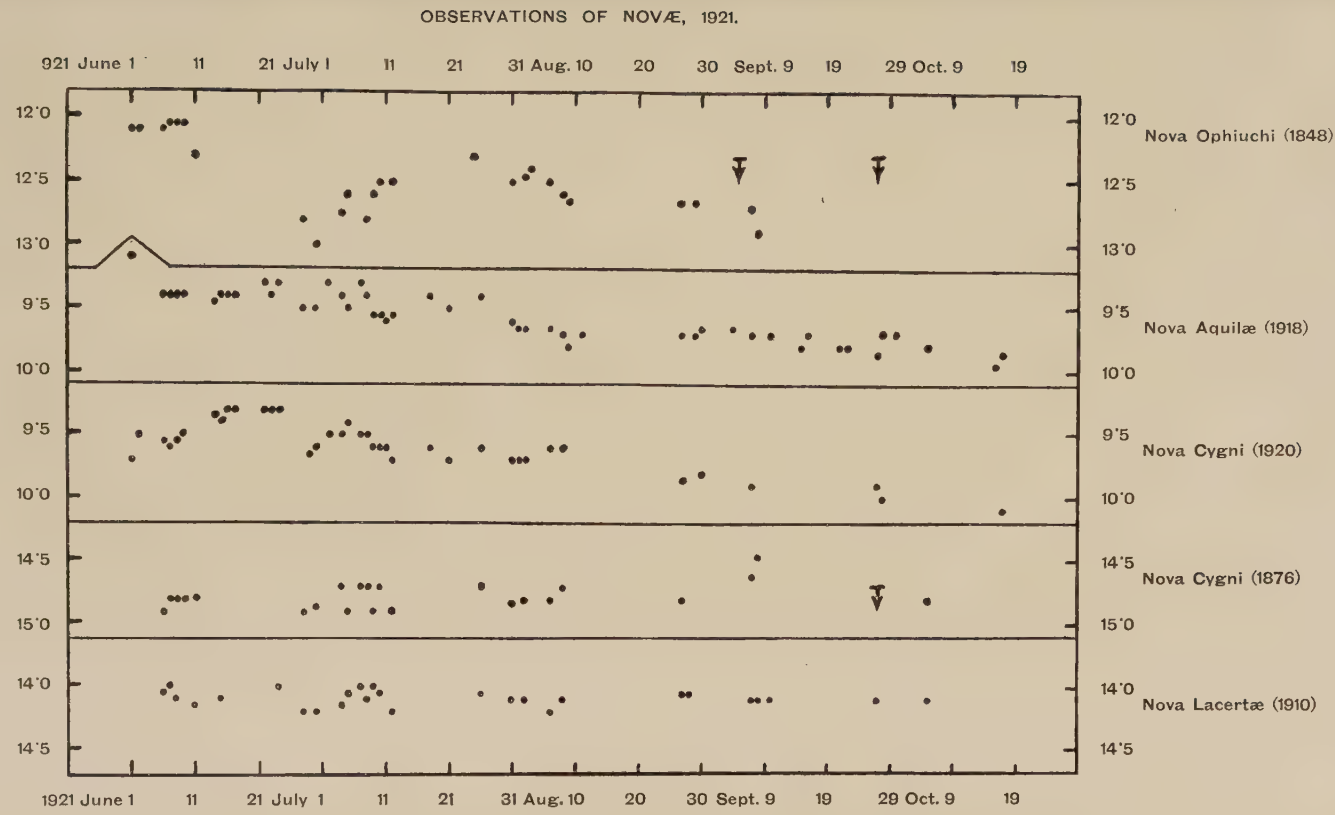
The magnitudes adopted for the comparison stars are those given by Barnard in the paper quoted above.

The focus for the Nova, as observed with G on June 11, appeared to be 2 or 3 mm. longer than for neighbouring stars, and I had the impression that, even when in focus, the image was not quite so well defined.

Nova Aquilæ (1918).

Observed on 49 nights between 1921 June 6 and Oct. 17. The star is still slowly fading, the mean magnitude having fallen from 9·4 to 9·9 during the period covered by the observations. Apart from minor oscillations of short period, which may or may not be real, the light curve is fairly smooth, though there is some slight evidence of a long-period variation, of small amplitude, whose length is of the order of 70 days.

Like certain other Novæ, the star is a difficult one to observe, being unduly large and bright to averted vision, especially when observed in a refractor. This is probably the combined effect of a slight difference of focus and the presence of nebulosity closely


OBSERVATIONS OF NOVÆ, 1921.
921 June 1
11
21 July 1
11
21
31 Aug. 10
20
30 Sept. 9
19
29 Oct. 9
19
12·0
12·5
13·0
9·5
10·0
9·5
10·0
14·5
15·0
14·0
14·5
Nova Ophiuchi (1848)
Nova Aquilæ (1918)
Nova Cygni (1920)
Nova Cygni (1876)
Nova Lacertæ (1910)
1921 June 1
11
21 July 1
11
21
31 Aug. 10
20
30 Sept. 9
19
29 Oct. 9
19

surrounding the object. When observed with G on June 11 the Nova appeared to be surrounded by a small nebulous haze, a few seconds in diameter. The focus seemed much the same as for other stars. The following estimates depend on a chart of comparison stars issued from Harvard:—

Date.	Mag.	Instrument.	Date.	Mag.	Instrument.
1921 June 1	9·1	S	1921 July 26	9·4	S
6	9·4	S	31	9·6	S
7	9·4	S	Aug. 1	9·65	S
8	9·4	S	2	9·65	S
9	9·4	S	6	9·65	S
14	9·45	S	8	9·7	S
15	9·4	S	9	9·8	S
16	9·4	S	11	9·7	S
17	9·4	S	27	9·7	S
22	9·3	S	29	9·7	S
23	9·4	S	30	9·65	S
24	9·3	S	Sept. 4	9·65	S
28	9·5	M	7	9·7	S
30	9·5	S	10	9·7	S
July 2	9·3	S	15	9·8	S
4	9·4	S	16	9·7	S
5	9·5	S	21	9·8	S
7	9·3	S	22	9·8	S
8	9·4	S	27	9·85	S
9	9·55	S	28	9·7	S
10	9·55	S	30	9·7	S
11	9·6	S	Oct. 5	9·8	S
12	9·55	S	16	9·95	M
18	9·4	S	17	9·85	M
21	9·5	S			

Nova Cygni (1920).

Observed on 38 nights between 1921 June 1 and Oct. 17. This star, like Nova Aquilæ, is still fading slowly. The mean magnitude fell from 9·4 to 10·0 during the period of the observations. There were slight, probably real, fluctuations of light between June 1 and July 12, but the subsequent estimates lie closely along a straight line. The Nova was not observed with G, but with S and M was, like Nova Aquilæ, particularly large and bright to averted vision. The magnitudes adopted for the comparison stars were determined by Mr. G. Merton, and distributed with a chart by the Director of the Variable Star Section of the B.A.A.

The individual estimates were as follows:—

Date.	Mag.	Instrument.	Date.	Mag.	Instrument.
1921 June 1	9·7	S	1921 July 8	9·5	
2	9·5	S	9	9·6	
6	9·55	S	10	9·6	
7	9·6	S	11	9·6	
8	9·55	S	12	9·7	
9	9·5	S	18	9·6	
14	9·35	S	21	9·7	
15	9·4	S	26	9·6	
16	9·3	S	31	9·7	
17	9·3	S	Aug. 1	9·7	
22	9·3	S	2	9·7	
23	9·3	S	6	9·6	
24	9·3	S	8	9·6	
28	9·65	M	27	9·85	
30	9·6	S	30	9·8	
July 2	9·5	S	Sept. 7	9·9	
4	9·5	S	27	9·9	
5	9·4	S	28	10·0	
7	9·5	S	Oct. 17	10·1	

Nova Cygni (1876).

Observed on 24 nights between 1921 June 6 and Oct. 5. The star was very near the limit of visibility with S and M, and was only visible in these instruments on particularly dark clear nights. The mean magnitude throughout the period was 14·81, and the estimates, with two exceptions, were within $0^{m}\cdot1$ of this value. When observed with G on June 11 no difference of focus or appearance as compared with other stars was noted. The estimate of magnitude (14·80) confirmed those made with the smaller instruments. The observations given below depend upon comparison stars whose magnitudes, recently determined by Professor Seares, are now used by Professor Barnard.

Date.	Mag.	Instrument.	Date.	Mag.	Instrument.
1921 June 6	14·91	S	1921 July 10	14·71	S
7	14·81	S	12	14·91	S
8	14·81	S	26	14·71	S
9	14·81	S	31	14·83	S
11	14·80	G	Aug. 2	14·81	S
28	14·91	M	6	14·81	S
30	14·87	S	8	14·71	S
July 4	14·71	S	27	14·81	S
5	14·91	S	Sept. 7	14·61	S
7	14·71	S	8	14·47	B and R
8	14·71	S	27	<14·71	S
9	14·91	S	Oct. 5	14·81	S

Nova Lacertæ (1910).

Observed on 26 nights between 1921 June 6 and Oct. 5. There appears to have been little or no change in the magnitude, which has been throughout within $0^{m}\cdot1$ of 14·1. The small oscillations in the early part of the curve are probably not real. No abnormality of focus or appearance could be detected with G on June 11. The seeing, however, on this night was not particularly good. The magnitudes of the comparison stars were taken from Barnard's list, published in the *Monthly Notices*, 72, 663.

Date.	Mag.	Instrument.	Date.	Mag.	Instrument.
1921 June 6	14·05	S	1921 July 10	14·05	S
7	14·0	S	12	14·2	S
8	14·1	S	26	14·05	S
11	14·15	G	31	14·1	S
15	14·1	S	Aug. 2	14·1	S
24	14·0	S	6	14·2	S
28	14·2	M	8	14·1	S
30	14·2	S	27	14·05	S
July 4	14·15	S	Sept. 7	14·1	S
5	14·05	S	8	14·1	P and R
7	14·0	S	10	14·1	S
8	14·1	S	27	14·1	S
9	14·0	S	Oct. 5	14·1	S

Nova Persei (1901).

Observations were commenced on 1921 Aug. 6. Up to Oct. 24 the star had been observed on 32 nights. The results so far show marked variability of an irregular type, the two extremes of brightness being 12·27 and 13·36, using Seares' magnitudes for the comparison stars (see *M.N.*, 80, 711).

As seen with G on Sept. 25, the Nova was indistinguishable from other stars in appearance, but there was a small patch of nebulosity, about 5″ in diameter, almost in contact with the star on its preceding side.

It is hoped to publish the details of the observations when a more extended series has been obtained.

The Solar Eclipse of 1921 *April* 7.
Summary of Observations.

(*Communicated by the Secretaries.*)

The following list gives the observers and their stations:—

No.	Observer.	Station.	Long. from Greenwich.	North Lat.	Source of Time.	Aperture, in.
			h m s	° ′ ″		
1	Rev. A. L. Cortie, S.J.	Stonyhurst	0 9 52·68 W.	53 50 40	Paris Wireless	
2	Rev. J. Rowland, S.J.	,,	,,	,,	,,	
3	Dr. A. A. Rambaut	Radcliffe Obs.	0 5 2·6 W.	51 45 36	Meridian Obsns.	
4	Mr. Robinson	,,	,,	,,	,,	10
5	Mr. Barrett	,,	,,	,,	,,	7
6	Mr. Balk	,,	,,	,,	,,	3¼
7	P. M. Ryves	Headley Obs., Rev. T. E. R. Phillips	0 1 4·3 W.	51 17	Paris and Nauen Wireless	8

Observed G.M.T. of Contacts.

Obs.	First Contact. h m s		Obs.	Last Contact. h m s	
2	19 38 22	*a, b*	2	22 6 23	*a*
4	19 34 55·6	*c*	4	22 4 59·7	*d*
5	19 34 55·0	*c*	5	22 4 55·5	*e*
6	19 34 55·3	*c*	5	22 4 58·0	*d*
			6	22 4 53·1	*f*
			7	22 4 20·6	*g*

Notes.

a. Observed by spectroscope. Slit radial at P.A. 252° and 63° respectively.
b. Signal given 5 seconds later, but corrected by estimation.
c. Moon's limb very rugged, and sun's somewhat unsteady; observation considered fairly satisfactory.
d. Disappearance of last mountain peak.
e. Disappearance of estimated mean limb of moon.
f. Probably early. Wind shook telescope.
g. Moon certainly visible 1ˢ before noted time.

Venus was clearly visible to the naked eye at Oxford and Stonyhurst, being held for 34ᵐ, up to 21ʰ 11ᵐ G.M.T., by Mr. Balk. No other stars or planets were seen, though looked for. The illumination at greatest phase was described as lurid and unnatural (Oxford), leaden or slaty, distinct from the colour of ordinary sunlight (Stonyhurst).

The passage of the moon over sunspots was watched by Mr. Robinson. No distortion or change in their appearance when close to the moon's limb could be detected.

Photographs.—Mr. Barrett took 15 photographs with 24-inch stopped to 4 inches. At Stonyhurst a plate exposed at 20ʰ 50ᵐ about greatest phase, with the Abney lens, focal length 34 inches, aperture stopped to

$f/100$, indicated a magnitude of 0·92. The photographic irradiation at each limb was found to be 0·03 mm. (about 7″).

The Campbell-Stokes sunshine recorder showed practically no trace from $20^h 41^m$ to $20^h 51^m$ (Oxford) and from $20^h 42^m$ to $21^h 4^m$ (Stonyhurst). The fall of temperature (from previous maximum) about greatest phase was 0°·4 F. (Oxford), 1° F. (Stonyhurst); it was noted as being considerably greater in the open air. At Stonyhurst there was no appreciable effect of the eclipse on the magnetic records.

The sky was practically cloudless at all the stations.

Mr. C. Potter observed Baily's Beads from Port Maddy, Benbecula, close to the central line. He used a 2½-inch refractor, and saw the Beads for some seconds before the end of annularity as solid round black globules moving upward along the limb of the sun, apparently raised above the moon's limb. He estimated their apparent diameter as 4″ to 8″, and expressed his opinion that they were not due to the lunar mountains.

Summary of Reports on the Occultation of Venus, 1921 *July* 1.

(*Communicated by the Secretaries.*)

The following list gives the observers and their stations:—

No.	Observer.	Station.	Long. from Greenwich.	North Lat.	Source of Time.	Aperture of Telescope.
			h m s	° ′ ″		in.
1	Sir F. W. Dyson	Greenwich	0 0 0	51 28 38	Greenwich obs.	4
2	A. C. D. Crommelin	,,	,,	,,	,,	6·7
3	C. R. Davidson	,,	,,	,,	,,	12
4	G. Merton	Woldingham	0 0 8·73 W.	51 17 51	Paris Wireless	6
5	W. H. Steavenson	W. Norwood	0 0 16·93 W.	51 26 14	,,	6
6	R. L. Waterfield	,,	,,	,,	,,	2·8
7	E. A. L. Attkins	Squirrel's Heath	0 0 50·4 E.	51 34 53	Hughes & Co., Fenchurch St.	8·5
8	Rev. A. L. Cortie, S.J.	Stonyhurst	0 9 52·68 W.	53 50 40	Paris Wireless	15
9	Rev. J. Rowland, S.J.	,,	,,	,,	,,	,,
10	E. H. Collinson	York	0 4 18 W.*	53 57*	Not stated*	4½

The asterisk indicates that great precision is not claimed for the corresponding data.

Observer 7 noted that his watch was 15^s slow July $0^d\ 23^h$, and 18^s slow July $1^d\ 23^h$: his times appear to require a correction of -2^s or -3^s.

Observed Times of Phases (G.M.T.).

Obs.	Disappearance.			Reappearance.	
	S. Cusp.	N. Cusp.	2L.	N. Cusp.	2L.
	h m s	s	s	h m s	s
1	16 2 ...	...	79·63	... ...	...
2	16 2 (44·32)	55·31	78·74	17 9 ...	28·53
3	16 2 51·36	55·35	77·78	... ...	...
4	16 2 32	...	57·3	... ...	...
5	16 2 ...	...	72·9	17 9 ...	20·0
6	16 2 ...	...	72·9	17 9 ...	18·0
7	16 3 (15)	...	35	17 9 28(*a*)	53(*a*)
8, 9	16 7 3·6	11·1	34·5	17 11 ...	25·1
10	16 7 42	...	78	17 12 ...	42
	Times reduced to Greenwich Observatory.				
4	16 2 53	...	78·3	... ...	...
5, 6	16 2 ...	...	78·4	17 9 ...	24·6
7	16 2 (60)	...	80	17 9 8	33

The times in brackets are presumably erroneous. Those marked (*a*) were noted by the observer as being probably 1 or 2 seconds late. This correction has not been applied above.

In reducing the times of the observers near London to what they would have been if made at Greenwich, the following data have been used, derived from *Circulaire de l'Observatoire de Cracovie*, No. 10:

Moving 1′ North makes Dis. 1ˢ·90 later, Re. 1ˢ·67 later.
,, 10ˢ East ,, ,, 0ˢ·57 later, ,, 1ˢ·85 later.

The calculated interval for Greenwich between the disappearance of S. cusp and 2L is 25ˢ·93; that between N. cusp and 2L is 21·37. Hence, collecting the observations of observers 1 to 6, and reducing to Greenwich and to 2L, we obtain:

Observer.	Time.	Object.
	h m s	
1	16 3 19·63	2L.
2	16·67	N.C.
,,	18·74	2L.
3	17·29	S.C
,,	16·72	N.C.
,,	17·78	2L.
4	18·9	S.C.
,,	18·3	2L.
5, 6	18·4	2L.
Weighted mean . .	16 3 18·23	(giving double weight to obs. of 2L).

The adopted time of reappearance of 2L at Greenwich is $17^h\ 9^m\ 28^s{\cdot}53$. At Stonyhurst the calculated intervals from disappearance of cusps to that of 2L are $28^s{\cdot}85$ S., $19^s{\cdot}45$ N.

The adopted times of disappearance and reappearance of 2L are $16^h\ 7^m\ 34^s{\cdot}0$, $17^h\ 11^m\ 25^s{\cdot}1$.

The following equations have been computed, using these adopted times. The *Nautical Almanac* R.A.'s of the moon and Venus are assumed to require corrections of $+x''$, $+e''$; the N.P.D.'s of $+y''$, $+f''$, the time of $+t^s$; the excesses of the tabular parallax and semidiameter of the moon over those of Venus are assumed to require multiplication by the factors $1+\frac{m}{1000}$, $1+\frac{n}{1000}$.

Dis. Gr.	$-14''{\cdot}95=+{\cdot}884(e-x)$	$-{\cdot}389(f-y)$	$-{\cdot}496\,t$	$-{\cdot}968\,m$	$-{\cdot}958\,n$	
,, St.	$-13{\cdot}88=+{\cdot}843$	,, $-{\cdot}478$	,, $-{\cdot}491$	,, $-{\cdot}529$	,, $-{\cdot}958$	,,
Re. Gr.	$+14{\cdot}36=-{\cdot}951$	,, $+{\cdot}135$	,, $+{\cdot}457$	,, $+1{\cdot}476$	,, $-{\cdot}959$	,,
,, St.	$+16{\cdot}03=-{\cdot}959$	,, $+{\cdot}011$	,, $+{\cdot}468$	,, $+1{\cdot}718$	,, $-{\cdot}959$	,,

An approximate solution, neglecting t, m, n, gives

$$(x-e)=+15''{\cdot}6,\quad (y-f)=-2''{\cdot}5:$$

the first is much better determined than the second.

Observers 3 and 10 both obtained photographs in broad daylight, some minutes before disappearance, on which the moon and Venus are distinctly visible.

Observers 8, 9 noted that at disappearance the S. cusp appeared brighter and sharper than the N. cusp. 7 contrasted the whiteness of Venus with the yellowness of the moon. 5 noted that some two seconds before the disappearance of Venus 2L the segment of the planet appeared greenish-blue and thickened at its ends (probably owing to diffraction). 10 noted the appearance of a dark shading inside the moon's limb, roughly making a circle when combined with the visible portion of the planet.

Occultations observed during the Lunar Eclipse of 1921 *October* 16.

At the lunar eclipse in 1921 October 16 the following occultations were observed at the Royal Observatory, Edinburgh (lat. 55° 55′ 30″·0, long. $12^m\ 44^s{\cdot}2$ west), by Professor Sampson (24-inch reflector) and Mr. R. W. Wrigley (15-inch refractor):—

Star.	Disapp.	Observer.	Reapp.	Observer.
B.D. 7°·218	10 15 21·9	R.A.S.		
224	10 18 8·4	R.A.S.	11 28 36·9	R.A.S.
	18 8·4	R.W.W.		
225	10 29 5·4	R.W.W.		
228	10 55 10·9	R.W.W.		
227	11 11 22·7	R.A.S.		
	11 11 22·4	R.W.W.		

The following occultations were observed at the Royal Observatory, Greenwich, by Mr. C. Davidson with the Merz equatorial ($12\frac{1}{4}$-inch):—

Star.	Disapp.
B.D. 7°·228	10 56 43·12
227	11 5 33·67

Eclipses of Jupiter's Satellites I. and II. observed at the Sydney Observatory in 1921.

(*Communicated by W. E. Cooke, Government Astronomer.*)

Disappearances. (*Start of Eclipse.*)

Satellite No.	Observed Time of Phenomenon. N.S.W. Standard Time.	G.M.T.	Phenomenon.
II.	Feb. 27th $9^h\ 22^m\ 22^s{\cdot}7$	Feb. 26th $23^h\ 22^m\ 22^s{\cdot}7$	Last speck at commencement of eclipse.

Appearances. (*Finish of Eclipse.*)

I.	Apr. 12th $10^h\ 14^m\ 53^s{\cdot}2$	Apr. 12th $0^h\ 14^m\ 53^s{\cdot}2$	First speck at finish of eclipse.
II.	Apr. 25th $8^h\ 54^m\ 23^s{\cdot}0$	Apr. 24th $22^h\ 54^m\ 23^s{\cdot}0$	Do.
II.	May 27th $8^h\ 31^m\ 59^s{\cdot}4$	May 26th $22^h\ 31^m\ 59^s{\cdot}4$	Do.

The observations were made by Mr. H. E. Rayner with the $11\frac{1}{4}$-inch Equatorial.

Observations of Comet 1921 *a* (*Reid*) *made at the University Observatory, Rostov-on-Don.* By Professor S. D. Tscherny.

(*Communicated by A. C. D. Crommelin.*)

[*Note.*—As Professor Tscherny had access only to a very limited collection of star catalogues, I have supplied the positions of several stars left blank by him, and revised the positions of other stars from more recent catalogues.—A. C. D. C.]

The observations were made with a 71-mm. Vion refractor, using a ring-micrometer; magnifying power 24. The observations are corrected for refraction, curvature of path, and motion of the comet. The observatory is in longitude $2^h\ 38^m\ 49^s{\cdot}48$ east of Greenwich, latitude $47^\circ\ 13'\ 9''{\cdot}4$ N.

G.M.T. 1921.		Comet—Star. R.A.	Comet—Star. Decl.	No. of Comps.	Log Factors of Parallax.		Comet's Apparent R.A.	Comet's Apparent Decl.	Star.
	d h m s	m s	′ ″				h m s	° ′ ″	
Apr.	30 7 30 30	+10 52·31	−30 25·1	1	9·8277*n*	0·7883	21 2 53·26	+ 60 57 39·2	*a*
May	8 7 7 17	+ 7 9·85	+ 6 54·2	4	9·7606*n*	0·8143	0 18 55·81	+ 84 37 46·8	*b*
,,	10 8 9 59	− 5 1·03	− 0 18·2	4	0·4045	0·7386	4 20 12·87	+ 84 50 40·6	*c*
,,	11 7 7 5	− 2 41·62	− 3 26·7	6	0·5161	0·4748	5 32 48·63	+ 83 31 32·2	*d*
,,	12 11 17 32	+ 1 54·17	+ 5 59·2	5	9·8799	0·8222	6 22 21·53	+ 81 28 54·9	*e*
,,	24 8 2 45	+ 4 46·91	+23 13·8	1	9·9456	0·6157	7 53 2·82	+ 63 56 5·8	*f*
,,	25 8 15 48	− 0 46·16	−24 32·6	4	9·9216	0·6594	7 54 41·08	+ 62 53 59·1	*g*
,,	26 7 23 45	− 0 0·32	+ 3 21·5	12	9·9267	0·5474	7 56 10·67	+ 61 54 27·0	*h*
,,	28 7 19 31	+ 4 2·50	+45 33·0	2	9·9006	0·5713	7 58 47·02	+ 60 1 19·5	*j*
,,	28 8 9 55	− 0 37·98	+31 51·8	5	9·8801	0·6852	7 58 49·27	+ 60 0 13·1	*k*
,,	31 7 58 22	+ 5 46·77	− 0 34·3	5	9·8503	0·6974	8 1 57·66	+ 57 29 0·9	*l*
,,	31 7 58 22	+ 3 14·09	−29 57·2	5	9·8505	0·6973	8 1 58·14	+ 57 29 55·4	*m*
June	1 8 11 54	− 4 12·62	+ 7 33·6	4	9·8321	0·7301	8 2 51·61	+(56 53 30·8)	*n*
,,	1 8 7 5	− 4 40·71	+ 7 3·3	5	9·8347	0·7225	8 2 53·05	+ 56 48 26·5	*o*
,,	3 7 54 0	+ 3 18 24	−14 26·2	3	9·8222	0·7201	8 4 30·09	+ 55 17 14·3	*p*
,,	3 7 54 0	+ 1 13·41	−12 16·0	3	9·8222	0·7201	8 4 30·18	+ 55 17 5·1	*q*
,,	3 7 54 0	− 0 6·76	− 4 54·6	3	9·8222	0·7201	8 4 30·96	+ 55 17 35·8	*r*
,,	4 9 45 22	+ 0 6·97	−15 12·4	5	9·8172	0·7161	8 5 15·91	+ 54 31 22·1	*s*
,,	4 9 45 22	+ 0 21·87	+ 3 24·5	5	9·8172	0·7161	8 5 16·45	+ 54 31 32·0	*t*
,,	5 7 54 10	+ 0 48·50	+33 54·8	4	9·8044	0·7382	8 5 55·49	+ 54 2 39·1	*u*
,,	6 8 6 33	+ 1 27·16		10	9·7850		8 6 34·13		*u*
,,	6 8 7 1		− 7 40·8	9		0·7659		+ 53 21 3·4	*u*
,,	8 8 43 40	+ 3 3·02		4	9·7251		8 7 49·30		*v*
,,	8 8 28 42		−13 44·2	3		0·8079		+ 52 8 50·1	*v*
,,	9 10 23 29	+ 5 56·92		3	9·4907		8 8 28·86		*w*
,,	9 10 37 53		−11 21·7	2		0·9162		+ 51 32 42·8	*w*
,,	11 9 2 48	+ 2 46·40	+16 51·2	1	9·6646		8 9 30·15	+ 50 31 31·7	*x*

Comparison Stars.

	Mean Place 1921·0. R.A.			Mean Place 1921·0. Decl.			Reduction to Apparent Place.		Authority.
	h	m	s	°	′	″	s	″	
a	20	51	59·71	+61	28	12·1	+1·24	−7·8	Helsingfors-Gotha 11735
b	0	11	47·69	+84	30	58·3	−1·73	−5·7	Grwh. 1900, 364, and Astrog. 47
c	4	25	17·52	+84	50	58·2	−3·62	+0·6	,, Astrog. 680
d	5	35	32·57	+83	34	56·7	−2·32	+2·2	,, ,, 560, ,, Astrog. 958
e	6	20	28·51	+81	22	52·9	−1·15	+2·8	,, ,, 989, ,, Astrog. 1334
f	7	48	15·42	+63	32	52·0	+0·49	0·0	Helsingfors-Gotha 5267
g	7	55	26·75	+63	18	31·7	+0·49	0·0	Boss P.G.C. 2106
h	7	56	10·47	+61	51	5·9	+0·52	−0·4	Helsingfors-Gotha 5338
j	7	54	43·98	+59	15	47·7	+0·54	−1·2	Boss P.G.C. 2101
k	7	59	26·70	+59	28	22·3	+0·55	−1·0	Kuffner 360, Hels.-Got. 5365
l	7	56	10·35	+57	29	37·1	+0·54	−1·9	Boss P.G.C. 2112
m	7	58	43·52	+57	59	54·3	+0·53	−1·7	,, ,, 2129
n	8	7	3·67	+56	45	59·1	+0·56	−1·9	Helsingfors-Gotha 5424
o	8	7	33·19	+56	41	25·1	+0·57	−1·9	Boss P.G.C. 2162
p	8	1	11·30	+55	31	43·0	+0·55	−2·5	Helsingfors-Gotha 5380
q	8	3	16·21	+55	29	23·6	+0·56	−2·5	,, ,, 5395
r	8	4	37·16	+55	22	32·9	+0·56	−2·5	,, ,, 5409
s	8	5	8·38	+54	46	37·2	+0·56	−2·7	Do. 5414 and Camb. U.S. 3056
t	8	4	54·01	+54	28	10·3	+0·57	−2·8	Camb. U.S. 3055.
u	8	5	6·41	+53	28	47·4	+0·58	−3·1	Grwh. Obs. 1920
,,		,,			,,		+0·56	−3·2	[Star-reduction, June 6]
v	8	4	45·72	+52	22	38·0	+0·56	−3·7	Paris 9948, Camb. U.S. 3054
w	8	2	31·37	+51	44	8·5	+0·57	−4·0	Boss P.G.C. 2145
x	8	6	43·16	+50	14	44·9	+0·59	−4·4	Camb. U.S. 3063

Notes.

May 10, 11, 28, 31.—Violent wind rendered observation difficult.
April 30; May 8, 12, 24, 25; June 4, 9, 11.—Clouds passing.
April 30; May 26, 31.—Ill-defined nucleus. May 25.—Well-defined nucleus.
June 5, 8, 9.—No condensation seen in coma.
June 6.—Condensation seen in coma.
June 5, 6.—Coma well defined.
April 30; May 12, 28, 31; June 1, 3, 4, 8, 9, 11.—Comet faint.
May 8.—Comet visible to naked eye, magnitude 5·5.
May 24.—Diameter of coma about 3′·5.
[June 1.—The first declination on this day is evidently erroneous.—A. C. D. C.]

Rostov-on-Don:
1921 *August* 18.